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A NEW ALGORITHM FOR ADAPTIVE SEQUENTIAL LOCATION OF DISTRIBUTION WAREHOUSES

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Abstract. Logistics can be defined as design, control and realization of material flows and together with them information flows that are developed by them. The basic aim of all logistics approaches is unremitting cost reduction. At the same time, it is necessary to improve customer services, especially their promptitude, quality, and availability.

One of the important tasks is movement of finished products from a producer to customers. If there is a need to supply a large–scale area with large number of customers, it is disadvantageous to deliver products from the only central warehouse or right from the producer. It is suitable to build up local distribution warehouses. The promptitude and quality of customer services improve as the number of warehouse increases. Freight cost created by transfer of products from a producer to customers is reduced too. On the other hand, both the fixed costs caused by the warehouse running and cost of maintaining of greater current, buffer and protective stocks increase all at once.

The contribution is concerned with a new adaptive sequential technique of design of the optimum network of distribution warehouses for supplying of selling centres. The basic criterion for decision is minimizing of the freight cost, but increasing expenses of product stocks are considered as well. This approach enables to increase the promptitude and quality of customer services and reduce overall cost, which leads in both cases to profit increase. The basic principles of the algorithm are explained on the model example of large–scale network of selling centres.

Key words. logistics strategy, distribution warehouses, location model, optimum location, optimum number of warehouses

AMS subject classifications. 90B05, 90B06, 90B80, 49M05

1. Introduction.

A problem of location of distribution warehouses as distribution channel elements was first formulated as early as in 1921 in connection with investigation of influence of both land prices and transportation cost on business. At present, choice of the optimum location of warehouses have to do with all elements of logistics channels where materials, intermediates and products are produced or stored and where the following element of a logistic channel (customer in a wide sense of the word) takes them over from the previous one (supplier in a wide sense of the word).

TABLE 1
Elements of logistics channels separated by stores.

Supplier	Customer
previous manufacturing stage	following manufacturing stage
store, plant	plant
producer, industrial distributor	producer, transporter
producer, transporter	wholesale
producer, wholesale	retail
producer, retail	household

Most frequently solved decision–making location problem is the problem of location of distribution warehouses. Basic models of location of distribution warehouses can be divided according to both methods of solution and their location in a distribution area into three patterns:

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- location by a market segment when the essential objective is completion of customer supplies,
- location by manufacturing principle when the warehouses are located as close as possible to the localities where the products are manufactured,
- combined location where the distribution warehouses are located between the productive and consumer centers by means of full cost analysis.

1.1. Problem of distribution–warehouse location. A number of exact mathematical methods, which are able to optimize location models, can be used for search of the optimum location of distribution warehouses. Determination of the optimum solution providing that market segments, which are to be supplied from one warehouse, are stated in advance is relatively simple standard mathematical problem. The optimum designation of market segments is considerably problematic and usually is replaced by partition of the supplied area into segments that are based on empirical geometric allocation or traditional business relations. Both approaches are simple and practicable without any difficulty but usually do not lead to the optimum allocation of the warehouses to the selling centres and consequently to creation of the optimum market segments.

Determination of the optimum market segment by means of full enumeration is not easy from the viewpoint of calculations because it is a NP–complete problem. Consumption of computer time increases faster than a polynomial (usually as much as a factorial) with increasing number of both distribution warehouses and selling centres supplied by them.

1.2. Problem formulation. A model of the optimum location of warehouses in connection with the optimally selected segments can be formulated as follows:

- n existing objects – selling centres P_i in a plane are given by their rectangular coordinates (x_i, y_i), $i = 1, 2, \ldots, n$,
- each selling centre has stated the size of a delivery q_i, $i = 1, 2, \ldots, n$, that has to be delivered within the given period from some of the warehouses,
- each selling centre P_i, $i = 1, 2, \ldots, n$, is allocated just into one of r market segments $\mathcal{T}_j$, $j = 1, 2, \ldots, r$ ($\mathrm{P}_i \in \mathcal{T}_j \iff t_i = j$), the allocation is not fixed but it can modify adaptively according to the situation in the course of the calculations,
- the number of the selling centres supplied by the distribution warehouse S_j is n_j, $j = 1, 2, \ldots, r$,
- the set $\mathcal{P}$ of the selling centres P_i is decomposed into r disjunctive subsets – bends of decomposition $\mathcal{T}_j$, which can represent just the market segments in the model,
- the system of the subsets $[\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_r]$ is called an ordered decomposition of the set of the selling centres $\mathcal{P}$ into the market segments if and only if:
  1. $\mathcal{T}_i \cap \mathcal{T}_j$ for $i \neq j$ (the segments are disjunctive),
  2. $\bigcup_{j=1}^{k} \mathcal{T}_j = \mathcal{P}$ (all the selling centres are included into the segments of the market),
  3. the order of the segments $\mathcal{T}_j$, $j = 1, 2, \ldots, r$, in the decomposition $[\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_r]$ is significant,
- each of the market segments $\mathcal{T}_j$ is supplied by one and only one distribution warehouse S_j,
- evidently $|\mathcal{T}_j| = n_j$, $j = 1, 2, \ldots, r$ and $n_1 + n_2 + \ldots + n_r = n$ are valid,

- the distance $\varrho_{ij},\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,r$, between the selling centres C_i and the distribution warehouses S_j is calculated as the straight Euclidean distance

$$\varrho_{ij}\left(x_i, y_i, X_j, Y_j\right) = \sqrt{\left(x_i - X_j\right)^2 + \left(y_i - Y_j\right)^2}, \tag{1}$$

  the distance can be corrected by the correction factor $k_{ij} \geq 1$ in case of need according to local conditions,
- the objective is to allocate r new objects – distribution warehouses $S_j,\ j = 1,2,\ldots,r$, in the plane and determinate consequently their rectangular coordinates $\left(X_j, Y_j\right),\ j = 1,2,\ldots,r$, in order to minimize the optimization criterion

$$\min_{(X_1,Y_1),\ldots,(X_r,Y_r)} \sum_{j=1}^{r} \sum_{i \epsilon \mathcal{T}_j} q_i \sqrt{\left(x_i - X_j\right)^2 + \left(y_i - Y_j\right)^2}. \tag{2}$$

1.3. Number of distribution warehouses. Solving of the problem of the optimum location of the distribution warehouses is considerably difficult considering the fact that explicit determination of the optimum number of the warehouses in the above model is not possible. According to the proposed method, first the optimum position of the only warehouse is calculated and then new warehouses are added systematically until further improvement of the objective obtained by adding another one is economically advantageous. The process is usually ended if small market segments begin to come into existence because it is disadvantageous to supply them from a separate distribution warehouse. The addition of another warehouse is increasingly expensive in principle. The common principle of decreasing profit of expended means manifests itself.

If the decomposition of the set of the selling centres $\mathcal{P}$ into r market segments $\mathcal{T}_j,\ j = 1,2,\ldots,r$, is done, the number of possible ordered decompositions of this set (containing n selling centres) is then given by the formula (evidently $n_1+n_2+\ldots+n_r = n$ is valid):

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1!\, n_2!\, \ldots\, n_r!}. \tag{3}$$

The number of possible decompositions of the number of selling centres n into r market segments supplied from the only distribution warehouse is given by the combination formula:

$$\binom{n+r-1}{n} \tag{4}$$

if stores supplying no selling centres are possible or

$$\binom{n-1}{r-1} \tag{5}$$

if each distribution warehouse supplies at least one selling centre. Considering the economic matter of the problem the market segments containing the only selling centre supplied from the only distribution warehouse can be excluded (economically disadvantageous – every further warehouse increases storage costs). A more complex

TABLE 2
Number of possible allocation of selling centres to distribution warehouses.

Number of distribution warehouses	Number of selling centres	Number of possible allocation
1	15	1
2	15	32 736
3	15	13 514 046
4	15	742 822 080
5	15	9 771 762 000
6	15	111 470 158 800
7	15	23 837 814 000
8	15	0

combination formula can be then derived by means of combination analysis. The results obtained from it are given in Tab. 2.

The global optimization is not practicable considering the problems connected with the explicit determination of the optimum number of the distribution warehouses. It is necessary to decompose the optimization problem and solve it sequentially as a system of optimization problems with increasing number of the warehouses. The system of the market segments is adaptively corrected twice: after the determination of the initial coordinates of the warehouses and then when the optimum coordinates of the warehouses are reached. It is the way of getting to the actual optimum for economically advantageous number of the warehouses.

The course of adaptive sequential determination of the optimum number of stores and consequently the optimum number of the market segments can be described in the following steps:

1. the optimum location of warehouses is determined first of all for the limit case of the only warehouse where it is no problem to allocate the selling centres to distribution warehouses,

2. in every following step another warehouse S_{r+1} is added which comes into existence by splitting of a selected warehouse $S_{r'}$ into two ones, this warehouse is selected by one of two possible ways:

- sequentially – as the last added warehouse ($r' = r$), the selling centre $P_{i_{max}}$ with the longest delivery distance is then determined by:

$$\varrho_{i_{max}r} = \max_{i=1,2,\ldots,n} \sqrt{(x_i - X_r)^2 + (y_i - Y_r)^2}, \tag{6}$$

- by means of a suitable heuristic rule:
  - as the warehouse, which supplies the remotest selling centre ($r' = j_{max}$), the distribution warehouse $S_{j_{max}}$ and selling centre $P_{i_{max}}$ with the longest delivery distance are determined by:

$$\varrho_{i_{max}j_{max}} = \max_{i=1,2,\ldots,n} \max_{j=1,2,\ldots,r} \sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2}, \tag{7}$$

  - as the warehouse which supplies most of the selling centres ($r' = j_{max}$), where

$$n_{j_{max}} = \max_{j=1,2,\ldots,r} n_j, \tag{8}$$

the selling centre $P_{i_{max}}$ with the longest delivery distance is then determined by:

$$(9) \qquad \varrho_{i_{max}j_{max}} = \max_{i=1,2,\dots,n} \sqrt{(x_i - X_{j_{max}})^2 + (y_i - Y_{j_{max}})^2},$$

– as the warehouse which has the greatest transportation costs, ($r' = j_{max}$), the distribution warehouse $S_{j_{max}}$ and selling centre $P_{i_{max}}$ with the greatest transportation costs are determined by:

$$(10) \quad q_{i_{max}} \varrho_{i_{max}j_{max}} = \max_{i=1,2,\dots,n} \max_{j=1,2,\dots,r} q_i \sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2},$$

3. the warehouse selected in the previous step is split into two warehouses (the number of the warehouses is increased by one) so that the first one of them is placed at the half distance in the direction to the selling centre $P_{i_{max}}$:

$$(11) \qquad X_{r'}^{new} = X_{r'} - (x_{i_{max}} - X_{r'})/2 \quad , \quad Y_{r'}^{new} = Y_{r'} - (y_{i_{max}} - Y_{r'})/2$$

and the second one S_{r+1} symmetrically in the opposite direction:

$$(12) \qquad X_{r+1} = X_{r'} + (x_{i_{max}} - X_{r'})/2 \quad , \quad Y_{r+1} = Y_{r'} + (y_{i_{max}} - Y_{r'})/2,$$

4. all the selling centres allocated up to now to the warehouse $S_{r'}$ are reallocated to one of the new warehouses $S_{r'}^{new}$ or S_{r+1} according to their Euclidean distance:

$$(13) \qquad \mathcal{T}_{r'}^{new} = \{C_i, \ i \in \mathcal{T}_{r'}, \ \varrho(x_i, y_i, X_{r'}, Y_{r'}) < \varrho(x_i, y_i, X_{r+1}, Y_{r+1})\},$$

$$(14) \qquad \mathcal{T}_{r+1} = \{C_i, \ i \in \mathcal{T}_{r'}, \ \varrho(x_i, y_i, X_{r+1}, Y_{r+1}) < \varrho(x_i, y_i, X_{r'}, Y_{r'})\},$$

5. the optimum location of the warehouses is carried out for the actual market segments,

6. the allocation of the selling centres to the distribution warehouses is adaptively corrected at the end of the location process,

7. it is advisable to check up the allocation items which are not completely unambiguous (the allocation items where the distance of the selling centres from the distribution warehouse is worse by about 15% of the maximum distance in the whole area where the selling centres and the distribution warehouses are placed)

$$(15) \qquad |\varrho_{it_i} - \varrho_{ij}| \le 0.15 \max_{k=1,2,\dots,r} \varrho_{ik}, \quad j = 1,2,\dots,r, \ i = 1,2,\dots,n \ ,$$

8. if the system of the selling–centre allocation $[\mathcal{T}_1, \mathcal{T}_2, \dots, \mathcal{T}_r]$ has been corrected in two previous steps, the optimum location of the warehouses has to be carried out once more,

9. the whole procedure is repeated until adding of the warehouses is economically advantageous considering the total costs.

1.4. Algorithm of optimum location of distribution warehouses. There are given n selling centres P_i by their rectangular coordinates (x_i, y_i), $i = 1,2,\dots,n$, in some area. In the same area, r market segments $\mathcal{T}_j$, $j = 1,2,\dots,r$, are determined too. Each of the selling centres is allocated into the only market segment and the allocation is known. Each of the market segments is supplied by the only warehouse S_j, $j = 1,2,\dots,r$. Each of the selling centres has assigned the size of a delivery

q_i, $i = 1, 2, \ldots, n$, which has to be delivered within the given period from some of the warehouses. The objective is to calculate such a location of the distribution warehouses in the area which minimizes the objective function:

$$\sum_{j=1}^{r} S_j\left(X_j, Y_j\right) = \sum_{j=1}^{r} \sum_{i \epsilon \mathcal{T}_j} q_i \sqrt{\left(x_i - X_j\right)^2 + \left(y_i - Y_j\right)^2}. \tag{16}$$

A set of equations can be obtained by differentiation of the objective function. Their solution gives the optimum coordinates of the warehouses (X_j, Y_j) , $j = 1, 2, \ldots, r$:

$$\frac{\partial S\left(X_j, Y_j\right)}{\partial X_j} = \sum_{i \epsilon \mathcal{T}_j} q_i \frac{x_i - X_j}{\sqrt{\left(x_i - X_j\right)^2 + \left(y_i - Y_j\right)^2}} \stackrel{!}{=} 0\ , \tag{17}$$

$$\frac{\partial S\left(X_j, Y_j\right)}{\partial Y_j} = \sum_{i \epsilon \mathcal{T}_j} q_i \frac{y_i - Y_j}{\sqrt{\left(x_i - X_j\right)^2 + \left(y_i - Y_j\right)^2}} \stackrel{!}{=} 0\ . \tag{18}$$

The conditional equations can be simply rewritten into a form suitable for iterative calculations (the iterations converge relatively fast to the optimum values, it is necessary to avoid possible singularities that can come into existence by zeroizing of the denominators of the equations), $k = 1, 2, \ldots,$:

$$X_j^{(k+1)} = \frac{\displaystyle\sum_{i \epsilon \mathcal{T}_j} \frac{q_i\ x_i}{\sqrt{\left(x_i - X_j^{(k)}\right)^2 + \left(y_i - Y_j^{(k)}\right)^2}}}{\displaystyle\sum_{i \epsilon \mathcal{T}_j} \frac{q_i}{\sqrt{\left(x_i - X_j^{(k)}\right)^2 + \left(y_i - Y_j^{(k)}\right)^2}}}\ , \quad j = 1, 2, \ldots, r\ , \tag{19}$$

$$Y_j^{(k+1)} = \frac{\displaystyle\sum_{i \epsilon \mathcal{T}_j} \frac{q_i\ y_i}{\sqrt{\left(x_i - X_j^{(k)}\right)^2 + \left(y_i - Y_j^{(k)}\right)^2}}}{\displaystyle\sum_{i \epsilon \mathcal{T}_j} \frac{q_i}{\sqrt{\left(x_i - X_j^{(k)}\right)^2 + \left(y_i - Y_j^{(k)}\right)^2}}}\ , \quad j = 1, 2, \ldots, r\ . \tag{20}$$

The weighted arithmetic averages can be chosen as initial approximations where the weights are given sizes of deliveries:

$$X_j^{(0)} = \frac{\displaystyle\sum_{i \epsilon \mathcal{T}_j} q_i\ x_i}{\displaystyle\sum_{i \epsilon \mathcal{T}_j} q_i}\ , \qquad Y_j^{(0)} = \frac{\displaystyle\sum_{i \epsilon \mathcal{T}_j} q_i\ y_i}{\displaystyle\sum_{i \epsilon \mathcal{T}_j} q_i}\ , \quad j = 1, 2, \ldots, r. \tag{21}$$

2. Application of algorithm. The procedure of searching for both the optimum number and location of the distribution warehouses is shown at a case study of a model example of a simple distribution network. There are placed 15 selling centres on an area of size of 200 km × 100 km. Their basic parameters:

- the positions given by means of two rectangular coordinates,
- the sizes of a delivery, which have to be delivered regularly in a given period

are shown in Tab. 3. All the parameters were generated by an modified generator of random numbers not to be distributed completely uniformly over the given area. In practice, selling centres occur in non–uniform clusters.

TABLE 3
Location of selling centers and their demands for supplies.

Selling center	Coordinate_1 [km]	Coordinate_2 [km]	Supplies [t]
1	82	53	1100
2	147	61	500
3	81	36	700
4	53	82	700
5	172	42	1200
6	51	85	1100
7	25	93	800
8	173	91	700
9	198	53	500
10	118	45	1400
11	68	53	1100
12	193	15	1300
13	40	99	1400
14	176	85	1400
15	126	22	1200

2.1. Course of warehouse location. The position of the first warehouse $S_{10}^{(1)}$ is estimated by means of both the equations (21). After optimizing its position, further distribution warehouse is created by splitting of the first warehouse $S_1^{(1)}$. Two new warehouses $S_{10}^{(2)}$ and $S_{20}^{(2)}$ come into existence. All the selling centers are allocated to them according the mutual distances. After further optimization of their position, the testing of possible modifications of the allocation of the selling centres is carried out.

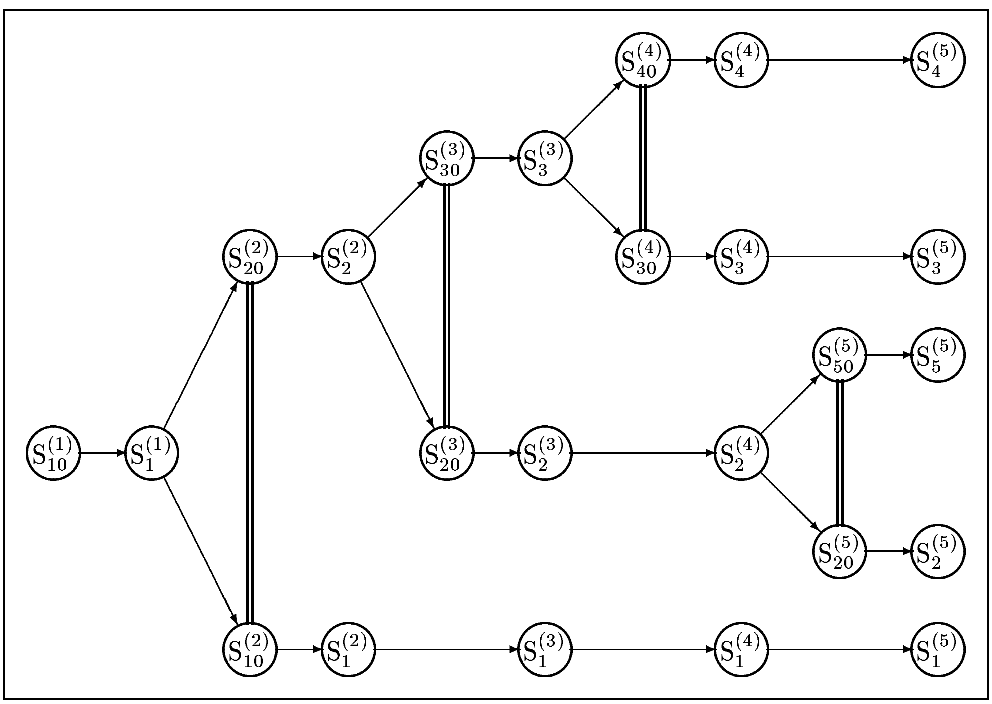

FIG. 1. *The chart of sequential addition of distribution warehouses.*

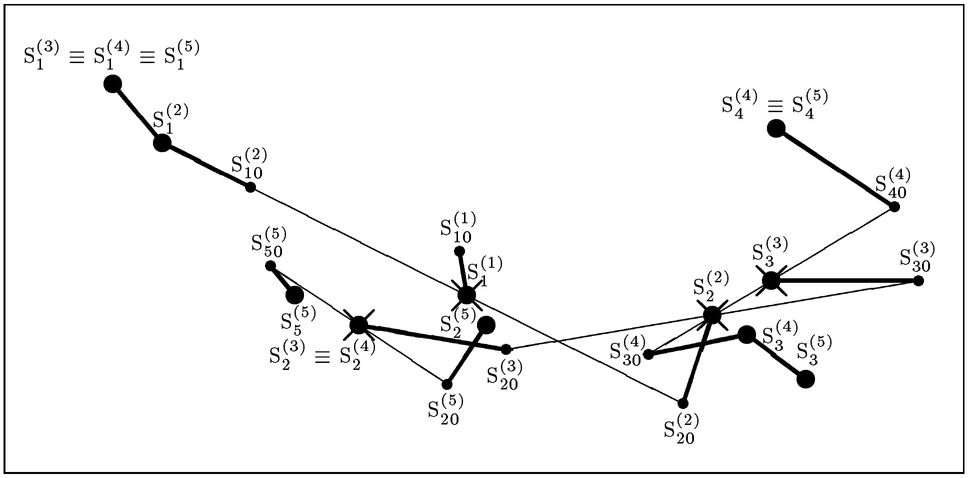

FIG. 2. *The modifications of the distribution–warehouse set in the course of their adding.*

In case that the value of the objective function is improved for some modification of the allocation the modification is accepted. The allocation of selling centres is adaptively modified according to the new positions of the distribution warehouses in every computational stage. The chart of sequential addition of further warehouses by splitting of one of the existing warehouses is shown in Fig. 1. The course of both changes of the location of the warehouses and addition of the warehouses in the supplied area are shown in Fig. 2. Splitting of a warehouse is marked by a cross; the transfer of both split warehouses is represented by thin lines. The process of the calculations of the optimum position of the warehouses is represented by thick lines.

2.2. Optimum location of five warehouses. During the optimum location of five distribution warehouses, the warehouse $S_2^{(4)}$ is split at first. After splitting this warehouse is moved from its actual position to a new one $S_{20}^{(5)}$ and the position

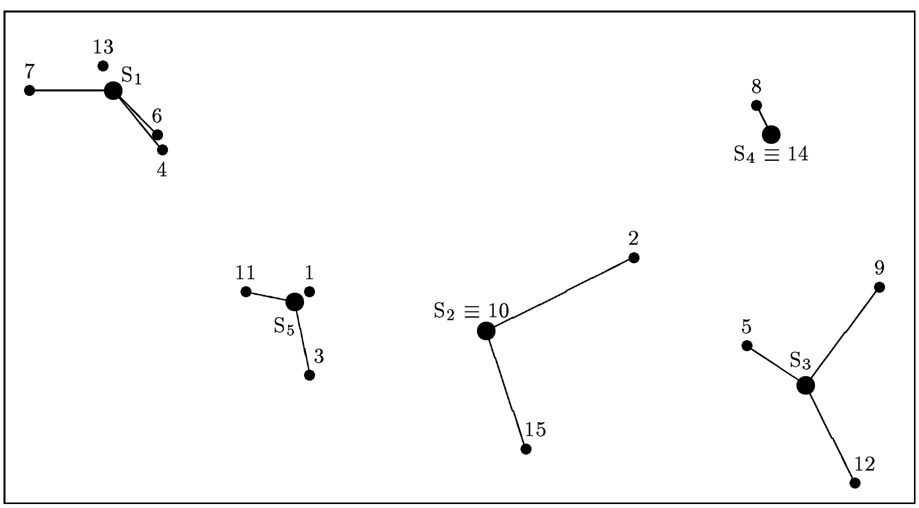

FIG. 3. *The optimum location of five distribution warehouses.*

TABLE 4
Allocation of selling centres to five warehouses.

Selling center	Distance from S_1 [km]	Distance from S_2 [km]	Distance from S_3 [km]	Distance from S_4 [km]	Distance from S_5 [km]	Allocation to warehouse
1	57,3	36,9	90,7	99,3	3,6	5
2	110,1	**33,1**	**31,4**	**37,6**	68,7	**3**
3	69,9	38,1	91,2	106,9	15,1	5
4	16,3	74,8	125,5	123,0	40,5	1
5	140,0	54,1	0,0	43,2	93,4	3
6	12,7	78,0	128,4	125,0	44,0	1
7	17,0	104,7	155,6	151,2	68,4	1
8	131,0	71,7	49,0	6,7	102,2	4
9	161,3	80,4	**28,2**	**38,8**	119,0	**3**
10	90,4	0,0	54,1	70,5	39,5	2
11	48,5	50,6	104,6	112,6	11,2	5
12	170,4	80,8	34,2	72,0	119,5	3
13	5,4	94,9	143,8	136,7	61,8	1
14	134,3	70,5	43,2	0,0	102,8	4
15	110,6	24,4	50,2	80,4	55,2	2

of a new warehouse $S_{50}^{(5)}$ is estimated. The optimum location of five distribution warehouses is shown in Fig. 3. The results of the process of allocation of the selling centres to the new warehouses are shown in Tab. 4. The possibilities of alternative adaptive modifications of the allocation are highlighted in the table. The acceptance or rejection of the proposed modifications are shown in Tab. 5.

TABLE 5
Testing of possible modifications in allocation of selling centres to five warehouses.

Allocation modification	Allocation of selling centres to warehouses	Value of criterion	Result of testing
—	5 3 5 1 3 1 1 4 3 2 5 3 1 4 2	181 616	Original
2 → S_2	5 **2** 5 1 3 1 1 4 3 2 5 3 1 4 2	180 232	Accepted
2 → S_4	5 **4** 5 1 3 1 1 4 3 2 5 3 1 4 2	182 493	Rejected
9 → S_4	5 2 5 1 3 1 1 4 **4** 2 5 3 1 4 2	186 917	Rejected
2 → S_2, 9 → S_4	5 **2** 5 1 3 1 1 4 **4** 2 5 3 1 4 2	184 357	Rejected
2 → S_4, 9 → S_4	5 **4** 5 1 3 1 1 4 **4** 2 5 3 1 4 2	186 618	Rejected

3. Conclusion. An original method of sequential decomposition of the problem of the optimum location of the distribution warehouses with adaptive modifications of the allocation of the selling centres into the market segments, which are represented by the distribution warehouses, is described in the article. The results summarized in Tab. 6 and Tab. 7 confirm the common principle of decreasing benefit of expended means. Every further warehouse, which is added to the set of warehouses, is relatively more expensive than the previous one.

TABLE 6
Improvement of optimization criterion depending on number of warehouses.

Number of warehouses	Value of criterion [tkm]	Absolute improvement [tkm]	Relative improvement [%]
1	879 348	—	—
2	478 629	400 719	45,57
3	349 698	128 931	14,66
4	263 044	86 654	9,85
5	180 232	82 812	9,42

TABLE 7
Course of sequential adaptive location of distribution warehouses.

Number of warehouses	Warehouse designation	Location of warehouse [km]		Supplying of selling centres	Size of delivery [t]	Contribution to criterion [tkm]	Value of criterion [tkm]
1	S_1	114	51	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	15 100	879 348	879 348
2	S_1	53	82	1 3 4 6 7 11 13	6 900	176 718	478 629
	S_2	165	47	2 5 8 9 10 12 14 15	8 200	301 911	
3	S_1	42	94	4 6 7 13	4 000	46 559	349 698
	S_2	91	45	1 3 10 11 15	5 500	137 507	
	S_3	176	52	2 5 8 9 12 14	5 600	165 632	
4	S_1	42	94	4 6 7 13	4 000	46 558	263 044
	S_2	91	45	1 3 10 11 15	5 500	137 507	
	S_3	172	42	2 5 9 12	3 500	74 283	
	S_4	176	85	8 14	2 100	4 696	
5	S_1	42	94	4 6 7 13	4 000	46 559	180 232
	S_2	118	45	2 10 15	3 100	45 782	
	S_3	183	34	5 9 12	3 000	56 338	
	S_4	176	85	8 14	2 100	4 696	
	S_5	79	51	1 3 11	2 900	26 857	
6	S_1	42	94	4 6 7 13	4 000	46 559	163 672
	S_2	118	45	10 15	2 600	29 222	
	S_3	183	34	5 9 12	3 000	56 338	
	S_4	176	85	8 14	2 100	4 696	
	S_5	79	51	1 3 11	2 900	26 857	
	S_6	147	61	2	500	0	

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