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NUMERICAL SOLUTION OF FLOW OVER A PROFILE CONSIDERING DYNAMICAL AND AEROELASTIC EFFECTS*

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Abstract. Numerical solution of a 2D inviscid incompressible flow over a profile in a channel.

Key words. model of Euler equations, finite volume method, prescribed oscillating behaviour, dynamical effects

1. Introduction. The work deals with a numerical solution of a 2D inviscid incompressible flow over a profile in a channel. The finite volume method in a form of cell-centered scheme is used. The composite scheme applied to a numerical solution consists of more dissipative part (Lax-Friedrichs scheme) and less dissipative part (Lax-Wendroff). Governing system of equations is the system of Euler equations. Two possibilities are considered. Firstly the flow is influenced by a prescribed oscillating behaviour of the profile. Secondly the oscillation of the profile is influenced by a flow field in the channel. In both cases the profile is fixed in the centre of gravity.

2. Mathematical model. The behaviour of flow is described by the system of Euler equations for inviscid incompressible flow in conservation form:

$$RW_t + F_x + G_y = 0 \ , \tag{1}$$

where

$$\begin{aligned} W &= \|\tfrac{p}{\rho}, u, v\|^T \\ F &= \|u, u^2 + \tfrac{p}{\rho}, uv\|^T \\ G &= \|v, uv, v^2 + \tfrac{p}{\rho}\|^T \\ R &= diag\|\tfrac{1}{a^2}, 1, 1\| \quad . \end{aligned} \tag{2}$$

Here ρ is density (constant), p is pressure and (u, v) is velocity vector. Upstream conditions are $W = W_\infty$. Downstream condition is only given $p = p_2$. Next values of W_2 are extrapolated. Wall conditions are nonpermeability conditions $(u, v)_n = 0$ (normal component of velocity vector is zero).

The *method of artificial compressibility* and the *time dependent method* are used for computation of the steady flow.

In the case of an unsteady flow one has to consider $a \to \infty$ or $a \gg K$, where K is a big positive number.

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The two possibilities of flow are considered:

- **Prescribed oscillating behaviour of the profile:**
  The change of a position (angle of attack) of the profile fixed in the centre of gravity is given (fig. 1) by the formula

$$\varphi = \varphi_0 \cos(2\pi f t) \ ,$$

where φ is an angle of rotation of the profile from the position of equilibrium $[rad]$,
φ_0 is an initial angle of rotation of the profile from the position of equilibrium at $t = 0$ $[rad]$,
f is a frequency $[s^{-1}]$, $f = 2\,\mathrm{Hz}$.

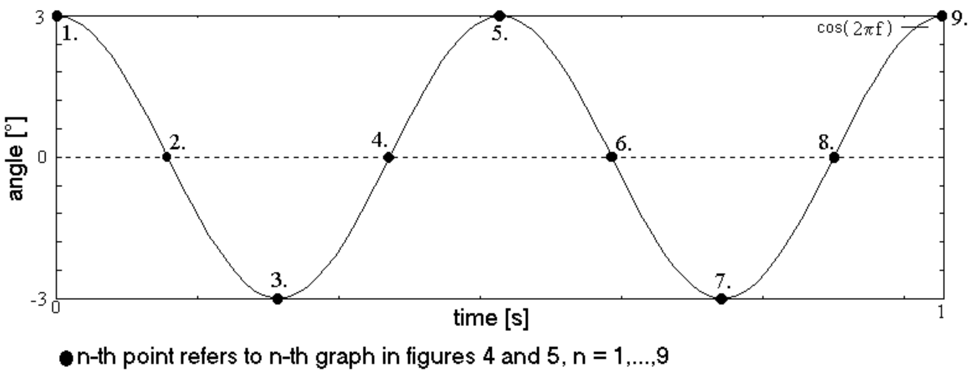

FIG. 1. *Angle of rotation of the profile (shows positions of the profile in graphs in fig. 4, 5)*

- **Dynamical effects of fluid flow on the profile:**
  Dynamical effects (changes of a position of the profile fixed in the centre of gravity) are considered by the behaviour of the ODE

$$\theta\ddot{\varphi} + k_t\dot{\varphi} + c_t\varphi = M_A(t) \ , \tag{3}$$

where φ is an angle of rotation of the profile from the position of equilibrium $[rad]$,
θ is a momentum of inertia $[kg\, m^2]$,
k_t is a coefitient of torsional damping $[kg\, m^2 s^{-1}]$,
c_t is a torsional stiffness $[kg\, m^2 s^{-2}]$,
$M_A(t)$ is an aerodynamic momentum.

3. Numerical solution. The combination of Lax-Wendroff (LW) and Lax-Friedrichs (LF) schemes in finite volume form of composite (C) scheme is used:

$$C(W) = LW(W) \otimes LF(W)$$

at quadrilateral grid (fig. 2) in cell-centered form:

- LF Predictor:

$$W_i^{n+1/2} = W_i^n - \frac{1}{2}\frac{\Delta t}{\mu_i}\sum_{k=1}^{4}(\bar{F}_{ik}^n \Delta y_k - \bar{G}_{ik}^n \Delta x_k) + \frac{\varepsilon}{4}\sum_{k=1}^{4}(W_k^n - W_i^n) \tag{4}$$

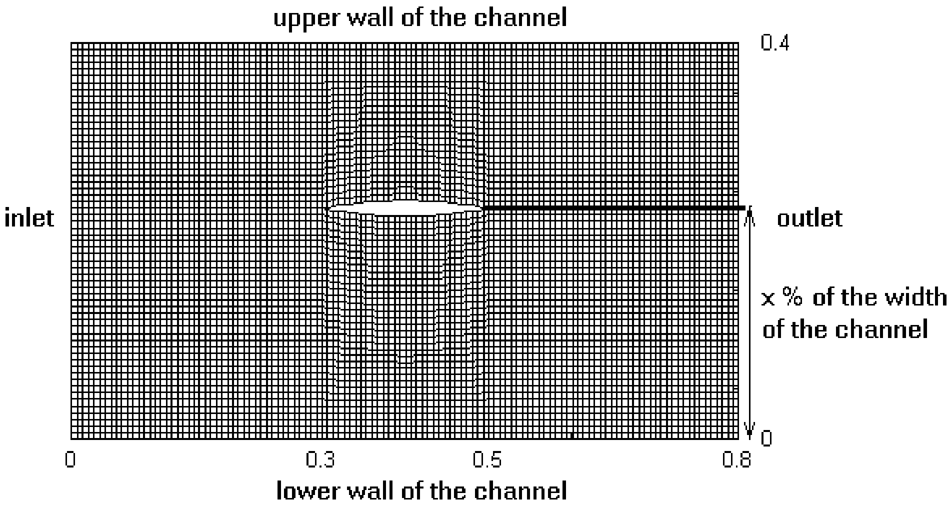

FIG. 2. *Structured mesh for a channel with a profile inside (Numerical results obtained for upstream conditions* $W = W_\infty = \|12500Pa, 5ms^{-1}, 0ms^{-1}\|$ *and downstream condition* $p = p_2 = 12500Pa$*)*

- LF Corrector:

$$W_i^{n+1} = W_i^{n+1/2} - \frac{1}{2}\frac{\Delta t}{\mu_i}\sum_{k=1}^{4}(\bar{F}_{ik}^{n+1/2}\Delta y_k - \bar{G}_{ik}^{n+1/2}\Delta x_k) +$$

$$+\frac{\varepsilon}{4}\sum_{k=1}^{4}(W_k^{n+1/2} - W_i^{n+1/2}) \tag{5}$$

- LW Predictor is the same as LF Predictor (4)
- LW Corrector:

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{\mu_i}\sum_{k=1}^{4}(\bar{F}_{ik}^{n+1/2}\Delta y_k - \bar{G}_{ik}^{n+1/2}\Delta x_k)\ , \tag{6}$$

where $\bar{F}_{ik} = \frac{1}{2}(F_k + F_{k+1}), \bar{G}_{ik} = \frac{1}{2}(G_k + G_{k+1}), \varepsilon \in (0, 1\rangle$ (fig. 3).

Wall conditions are realised by using "reflection principle", i. e. there are artificial volumes in the wall created by a reflection on the wall. The velocity vector (u, v) is also reflected on the wall.

Wall conditions on oscillating profile are realised by using "small disturbance theory" because one considers only small changes of angle φ, i. e. there is no real movement of the profile, the movement is approximated by the rotation of the normal vector to the profile ($\frac{\partial(u,v)}{\partial s} = f' + \varphi$, where f is a function describing the pofile, s is a tangential vector to the profile and $\varphi \leq 8^0$).

The ODE (3) is solved by the Runge-Kutta method of 4^{th} order.

4. Some numerical results. Figure 2 shows the channel with the profile inside and the structured quadrilateral mesh. The value x denotes the vertical profile position in the channel with respect to lower wall of the channel. Figures 1, 4 and 5

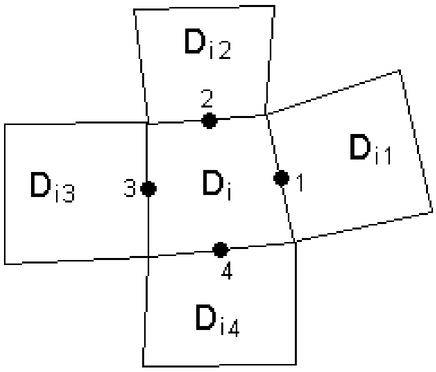

FIG. 3. *Using of neighbouring cells by application of numerical schemes in cell-centered form*

concerns the prescribed oscillating behaviour of the profile. The figure 1 represents a distribution of the profile oscillation prescribed as the function $\varphi = \varphi_0 \cos(2\pi f t)$, where $f = 2\,\text{Hz}$. There are considered small angles of profile rotation up to $\varphi_0 = 8$ degrees. In the case of results presented is $\varphi_0 = 3^o$. Figure 4, resp. 5 shows graphs of velocity value $q = \sqrt{u^2 + v^2}$ distributed along walls of the profile or along upper or lower wall of the channel. The figures are numbered with respect to numbers in fig. 1. It means that the n^{th} graph shows flow field in the corresponding time marked n in fig. 1. Charts labeled with number 1 are steady states of flow. After a transition state there it is possible to see periodical changes of flow with respect to periodical movement of the profile in figures 4, 5. Figure 6, resp. 7 represents oscillation of the profile described by the equation 3 for different upstream velocities u_∞ with profile position $x = 58,3\%$, resp. $x = 83,3\%$. Damping ratio [in %] depended on upstream velocity value is displayed in figure 8 for profile position $x = 58,3\%$, resp. figure 9 for profile position $x = 83,3\%$.

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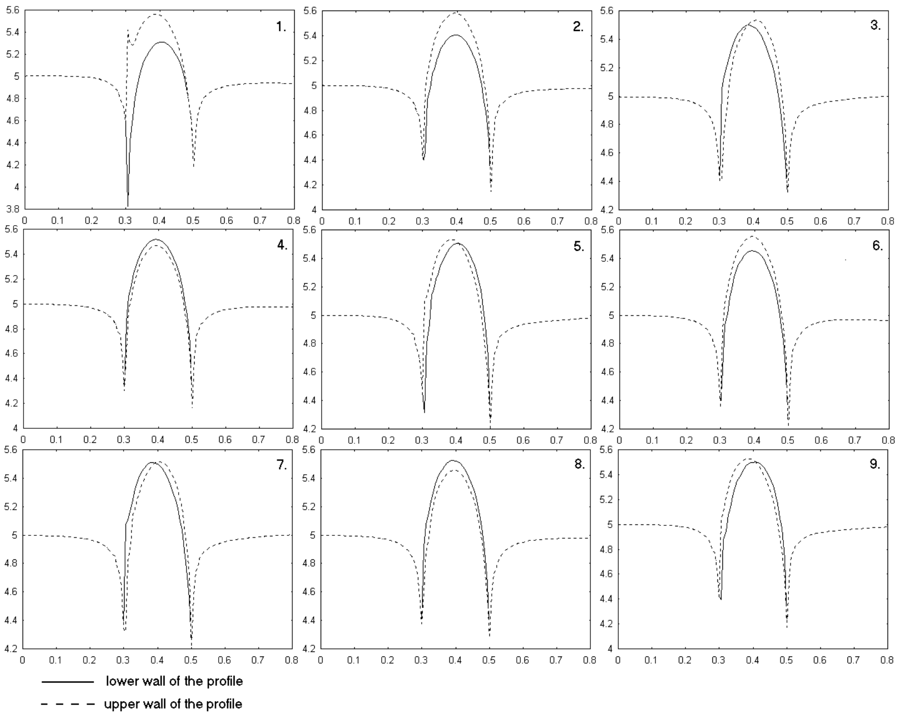

FIG. 4. *Velocity value distributed along the walls of the profile*

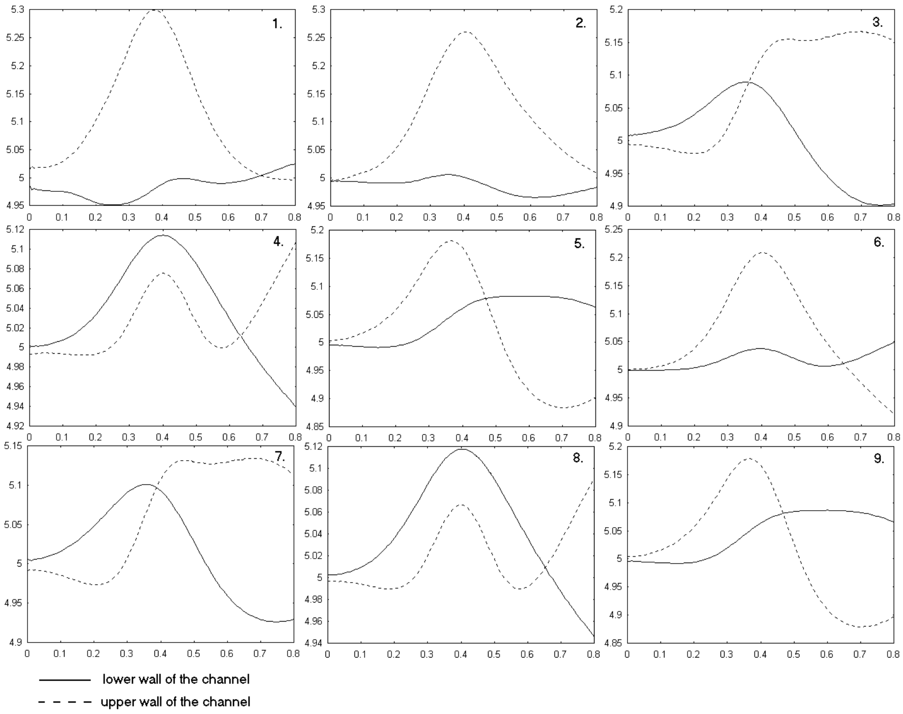

FIG. 5. *Velocity value distributed along upper and lower wall of the channel*

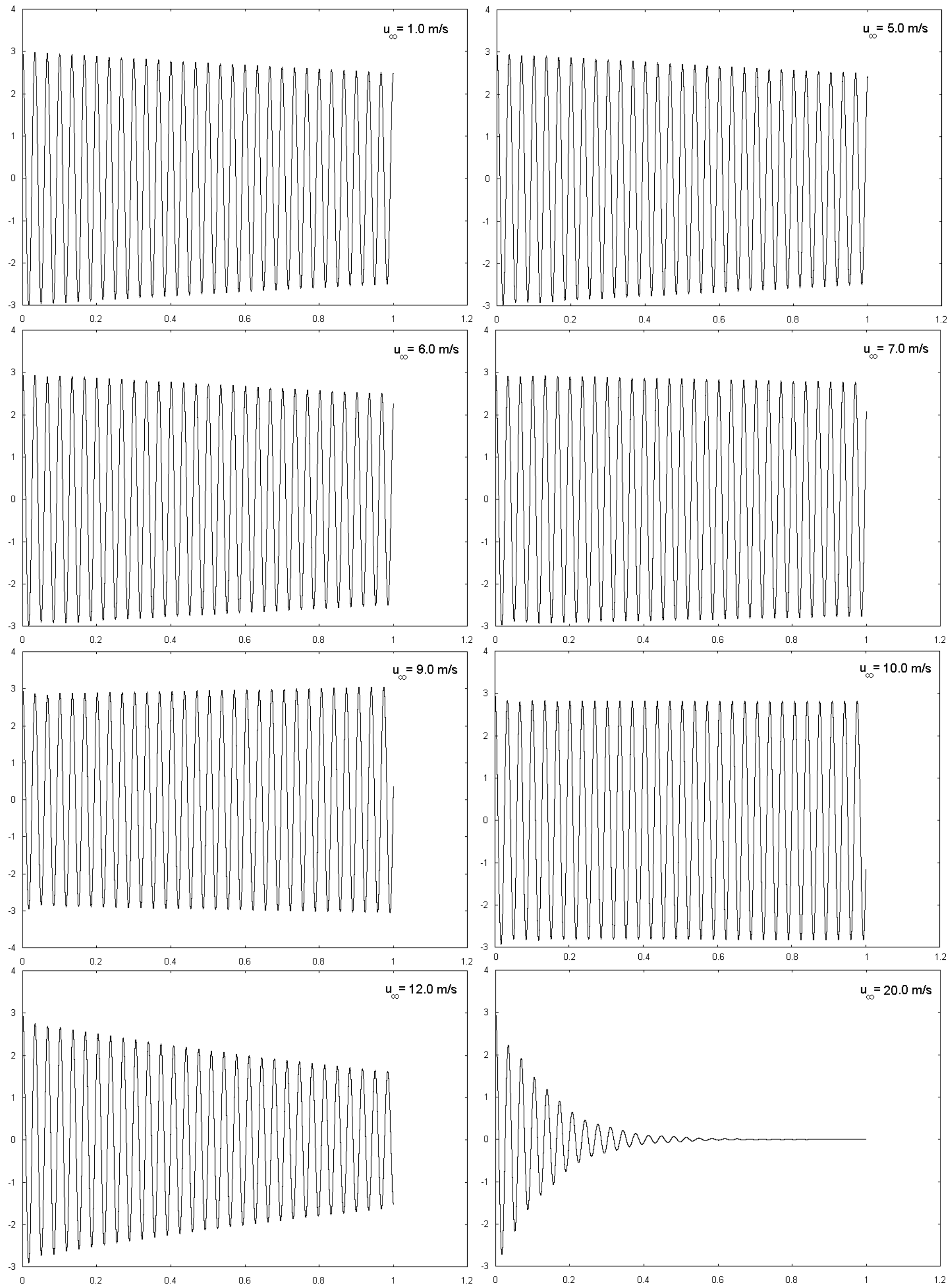

FIG. 6. *Oscillation of the profile with respect to different upstream velocities* u_∞, *position of the profile is* $x = 58.3\%$ *(fig. 2)*

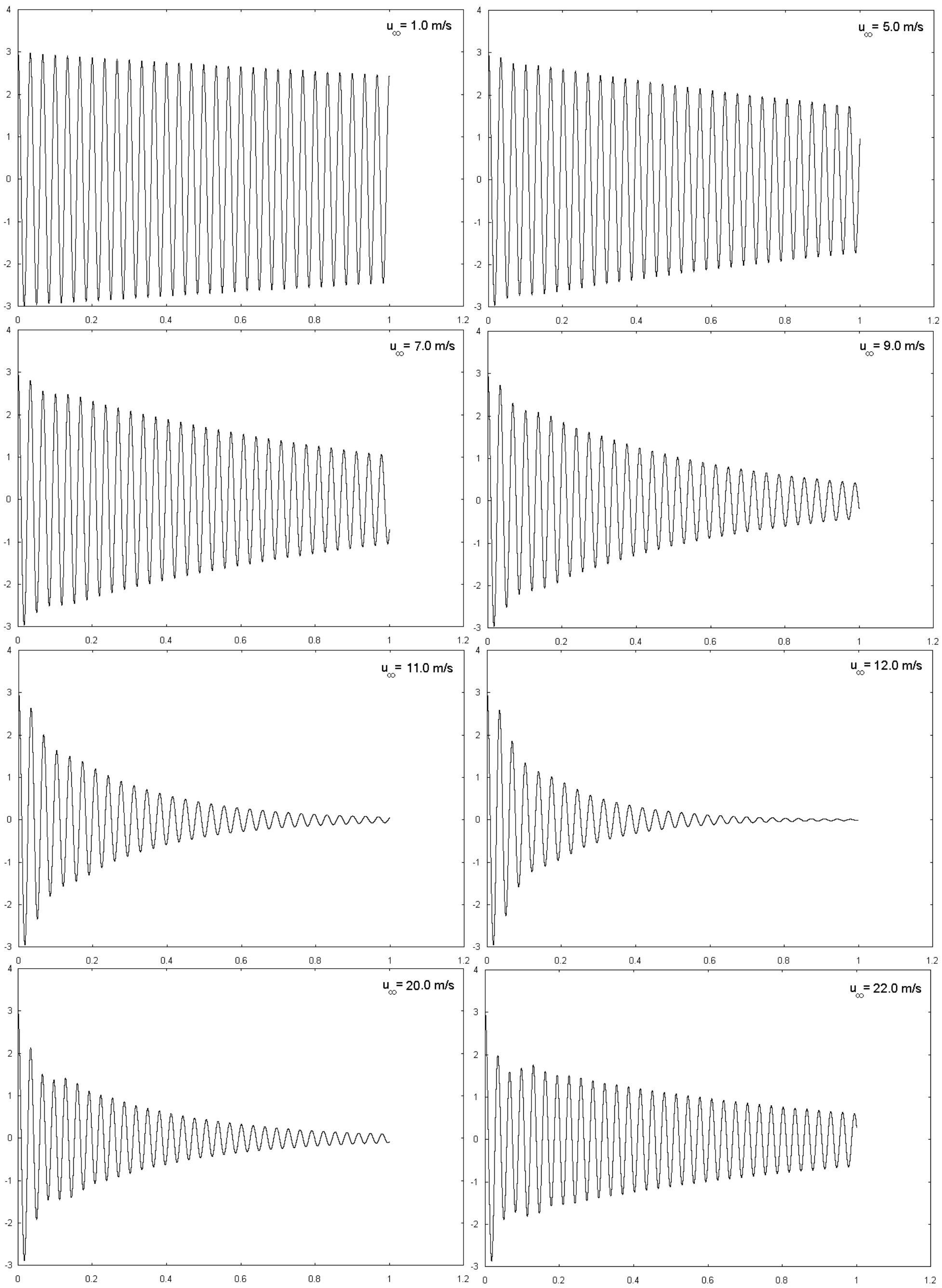

FIG. 7. *Oscillation of the profile with respect to different upstream velocities* u_∞*, position of the profile is* $x = 83.3\%$ *(fig. 2)*

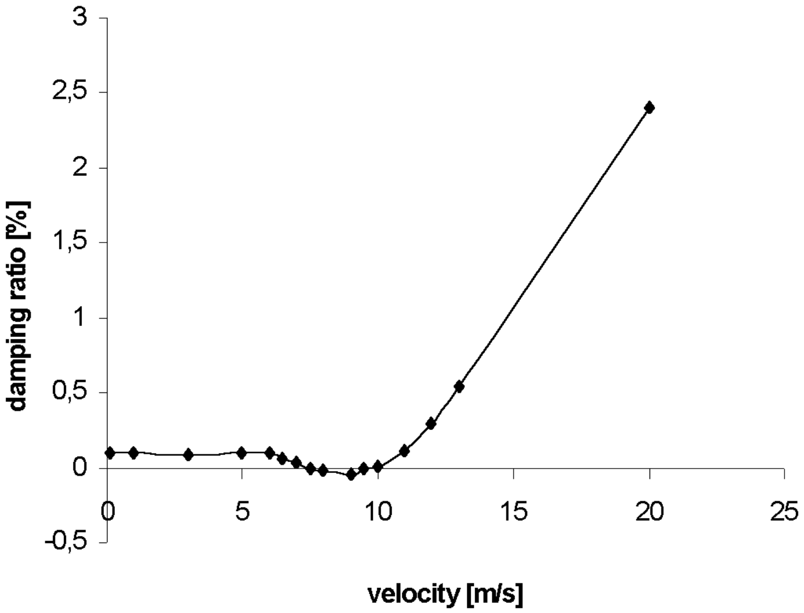

FIG. 8. *Damping ratio with respect to upstream velocity, position of the profile is* $x = 58.3\%$ *(fig. 2)*

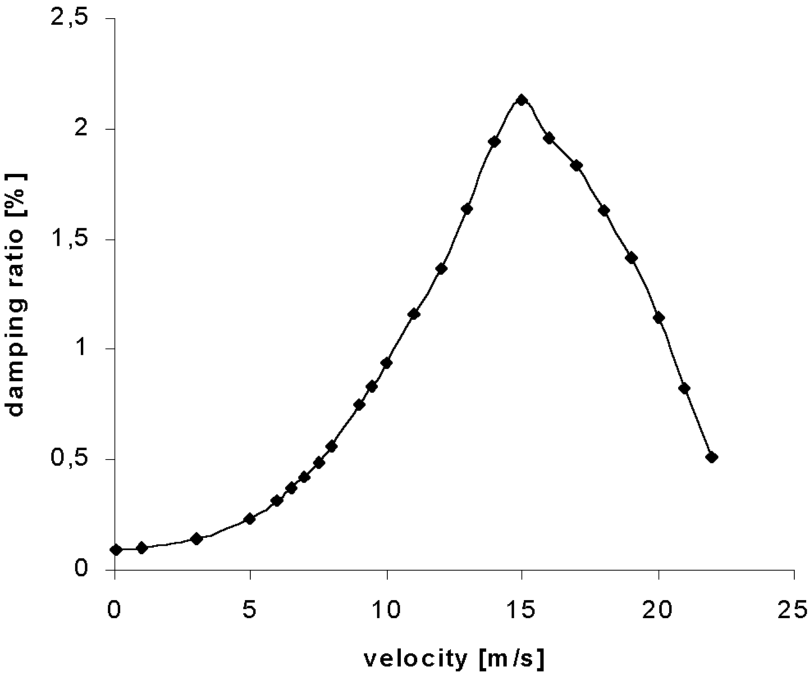

FIG. 9. *Damping ratio with respect to upstream velocity, position of the profile is* $x = 83.3\%$ *(fig. 2)*