Proceedings of ALGORITMY 2009
pp. 256–265

NONLINEAR INTERACTION OF INCOMPRESSIBLE FLOW AND A VIBRATING AIRFOIL WITH THREE DEGREES OF FREEDOM*

MARTIN RŮŽIČKA†, MILOSLAV FEISTAUER‡, JAROMÍR HORÁČEK§, AND PETR SVÁČEK¶

Abstract. The subject of the paper is the numerical simulation of the interaction of two-dimensional incompressible viscous flow and a vibrating airfoil with large amplitudes. A solid airfoil with three degrees of freedom performs rotation around an elastic axis and oscillations in the vertical direction and rotation of a flap. The numerical simulation consists of the finite element solution of the Navier-Stokes equations coupled with a system of ordinary differential equations describing the airfoil motion. The time-dependent computational domain and a moving grid are treated by the Arbitrary Lagrangian-Eulerian formulation of the Navier-Stokes equations. High Reynolds numbers require the application of a suitable stabilization of the finite element discretization.

Key words. aero-elasticity, flutter, Navier-Stokes equations, Arbitrary Lagrangian-Eulerian formulation, finite element metod, stabilization for high Reynolds numbers

AMS subject classifications. 65M60, 76M10, 76D05

1. Introduction. The interaction of fluids and structures plays an important role in many fields of science and technology. The research in aero-elasticity or hydro-elasticity focuses on the interaction between flowing fluid and vibrating structures. The aero-elastic stability of aerospace vehicles and the aero-elastic responses represented by dynamic load prediction and vibration levels in wings, tails and other aerodynamic surfaces have a great impact on the design as well as in the cost and operational safety.

In the contribution we consider a two-dimensional viscous incompressible flow past a moving airfoil, which is considered as a solid flexibly supported body with three degrees of freedom, allowing its vertical and torsional oscillations and the rotation of a flap. The numerical simulation consists of the finite element solution of the Navier-Stokes equations coupled with the system of ordinary differential equations describing the airfoil motion. The time dependent computational domain and a moving grid are taken into account with the aid of the Arbitrary Lagrangian-Eulerian (ALE) formulation of the Navier-Stokes equations.

In order to avoid spurious numerical oscillations, the SUPG and div-div stabilization is applied. The solution of the ordinary differential equations is carried out by the Runge-Kutta method. Special attention is paid to the construction of the ALE mapping of a reference domain on the computational domain at individual time

*This work was supported by Grant No. 48607 of the Grant Agency of the Charles University in Prague (M. Růžička), Grant No. 201/08/0012 of the Grant Agency of the Czech Republic (M. Feistauer, P. Sváček) and Grant No. IAA 200760613 of the Grant Agency of the Academy of Sciences of the Czech Republic (J. Horáček).

†Charles University Prague, Faculty of Mathematics and Physics, Sokolovská 83, 186 75 Praha 8, Czech Republic (mart.in.ruza@seznam.cz).

‡Charles University Prague, Faculty of Mathematics and Physics, Sokolovská 83, 186 75 Praha 8, Czech Republic (feist@karlin.mff.cuni.cz).

§Institute of Thermomechanics of the Academy of Sciences of the Czech Republic, Dolejškova 5, 182 00 Praha 8, Czech Republic (jaromirh@it.cas.cz).

¶Czech Technical University Prague, Faculty of Mechanical Engineering, Karlovo n. 13, 121 35 Praha 2, Czech Republic (svacek@marian.fsik.cvut.cz).

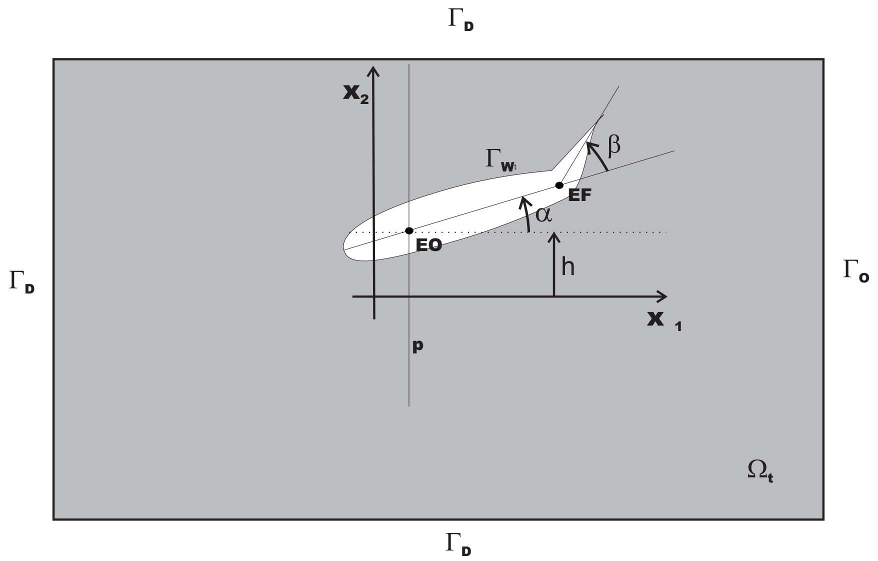

FIG. 2.1. *Model scheme*

instances. The resulting nonlinear discrete algebraic system is solved by the Oseen iterative process. All components of the realization of the solution are assembled together by a coupling procedure. As the result we obtained a sufficiently accurate and robust method for the numerical simulation of flow induced vibrations of an airfoil. The method was tested on a problem for which the results computed in NASTRAN program code are available. The comparison of our computations and the NASTRAN results shows good agreement.

2. Mathematical model. The two-dimensional non-stationary flow of viscous, incompressible fluid is considered in the time interval $[0, T]$, where $T > 0$. The symbol Ω_t denotes the computational domain occupied by the fluid at time t. The boundary $\partial\Omega_t = \Gamma_D \cup \Gamma_O \cup \Gamma_{W_t}$, where the sets Γ_D, Γ_O a Γ_{W_t} are mutually disjoint and boundary conditions of different types are used there. The symbol Γ_D represents the inlet, where the fluid flows into the domain Ω_t, or a fixed, impermeable wall. Γ_O represents the outlet, where the fluid leaves Ω_t and Γ_{W_t} is the moving airfoil boundary at time t. We assume that Γ_D and Γ_O are independent of time in contrast to Γ_{W_t}. The flow is characterized by the velocity field $\boldsymbol{u} = \boldsymbol{u}(x, t)$, and the kinematic pressure $p = p(x, t)$, for $x \in \Omega_t$ and $t \in [0, T]$. The kinematic pressure is defined as P/ρ, where P is the pressure and $\rho = \text{const.} > 0$ is the density of the fluid. The aim is to find functions $\alpha(t)$, $\beta(t)$ and $h(t)$, describing the rotation of the whole airfoil Γ_{W_t}, the rotation of the flap K of the airfoil and the vertical displacement of the airfoil, respectively. Hence, the shape of the domain Ω_t depends on the functions $\alpha(t)$, $\beta(t)$ and $h(t)$ as shown in Figure 2.1, where EO represents the elastic axis of the whole airfoil and EF represents the elastics axis of the flap. The elastic axis EO can move along the line p.

2.1. ALE formulation of the Navier-Stokes equations. In order to take into account the dependence of the computational domain on time, we introduce the ALE (Arbitrary Lagrangian–Eulerian) description, which is based on a smooth, one-to-one mapping

$$\boldsymbol{A}_t : \Omega_0 \mapsto \Omega_t, \quad \boldsymbol{X} \mapsto x(\boldsymbol{X},t) = \boldsymbol{A}_t(\boldsymbol{X}), \quad t \in [0,T]. \tag{2.1}$$

$\boldsymbol{A}_t$ is the identity in the part of the boundary $\partial\Omega_t$, where there is no interaction with the body and also there is no deformation of the boundary. The reference domain Ω_0 is identical with the domain occupied by the fluid at the initial time $t = 0$. The coordinates of points $x \in \Omega_t$ are called spatial coordinates, the coordinates of points $\boldsymbol{X} \in \Omega_0$ are called ALE coordinates or reference coordinates.

First, we define the domain velocity

$$\widetilde{\boldsymbol{w}}(\boldsymbol{X},t) = \frac{\partial}{\partial t} x(\boldsymbol{X},t). \tag{2.2}$$

This velocity can be expressed in the spatial coordinates as

$$\boldsymbol{w} = \widetilde{\boldsymbol{w}}(\boldsymbol{X},t) \circ \boldsymbol{A}_t^{-1}, \quad \text{i.e.} \quad \boldsymbol{w}(x,t) = \widetilde{\boldsymbol{w}}\left(\boldsymbol{A}_t^{-1}(x),t\right). \tag{2.3}$$

Let us consider a function $f = f(x,t), x \in \Omega_t, t \in [0,T], f(x,t) \in I\!R$, where $I\!R$ is the set of real numbers. Let us set $\widetilde{f}(\boldsymbol{X},t) = f(\boldsymbol{A}_t(\boldsymbol{X}),t)$. We define the ALE derivative of the function f by

$$\frac{D^A}{Dt} f(x,t) = \frac{\partial \widetilde{f}}{\partial t}(\boldsymbol{X},t), \quad \boldsymbol{X} = \boldsymbol{A}_t^{-1}(x). \tag{2.4}$$

The application of the chain rule gives

$$\frac{D^A}{Dt} f = \frac{\partial f}{\partial t} + \boldsymbol{w} \cdot \nabla f. \tag{2.5}$$

Using the relation (2.5), we obtain the Navier-Stokes equations in the ALE form

$$\begin{array}{rl} \frac{D^A}{Dt}\boldsymbol{u} + \left[(\boldsymbol{u}-\boldsymbol{w})\cdot\nabla\right]\boldsymbol{u} + \nabla p - \nu\Delta\boldsymbol{u} & = 0 \\ \operatorname{div}\boldsymbol{u} & = 0 \end{array} \quad \text{in } \Omega_t. \tag{2.6}$$

The symbol ν is a positive constant denoting the kinematic viscosity of the fluid.

2.2. Equations for the moving airfoil. The nonlinear equations of the motion describing the vibrations of the airfoil given by the functions α, β and h read

$$\begin{array}{ll}
m\ddot{h} + \left[(S_\alpha - S_\beta)\cos\alpha + S_\beta\cos(\alpha+\beta)\right]\ddot{\alpha} & \\
+S_\beta\cos(\alpha+\beta)\ddot{\beta} - (S_\alpha - S_\beta)\sin\alpha\dot{\alpha}^2 & \\
-S_\beta\sin(\alpha+\beta)(\dot{\alpha}+\dot{\beta})^2 + D_{hh}\dot{h} + k_{hh}h & = \mathcal{L}, \\
 & \\
\left[(S_\alpha - S_\beta)\cos\alpha + S_\beta\cos(\alpha+\beta)\right]\ddot{h} & \\
+\left[(I_\alpha - 2x_{1T}S_\beta) + 2x_{1T}S_\beta\cos\beta\right]\ddot{\alpha} & \\
+\left[I_\beta + x_{1T}S_\beta\cos\beta\right]\ddot{\beta} - x_{1T}S_\beta\sin\beta\dot{\beta}^2 & \\
-2x_{1T}S_\beta\sin\beta\dot{\alpha}\dot{\beta} + D_{\alpha\alpha}\dot{\alpha} + k_{\alpha\alpha}\alpha & = \mathcal{M}_\alpha, \\
 & \\
S_\beta\cos(\alpha+\beta)\ddot{h} + \left[I_\beta + x_{1T}S_\beta\cos\beta\right]\ddot{\alpha} & \\
+I_\beta\ddot{\beta} + x_{1T}S_\beta\sin\beta\dot{\alpha}^2 + D_{\beta\beta}\dot{\beta} + k_{\beta\beta}\beta & = \mathcal{M}_\beta.
\end{array} \tag{2.7}$$

For the derivation, see [4]. The symbol $\mathcal{L}$ stands for the component of the force acting on the whole profile in the vertical direction, $\mathcal{M}_\alpha$ is the torsional moment of the force acting on the whole airfoil with respect to the axis EO, $\mathcal{M}_\beta$ is the torsional moment of the force acting on the flap of the airfoil with respect to the flap axis EF, D_{hh}, $D_{\alpha\alpha}$, $D_{\beta\beta}$ are the coefficients of a structural damping, S_α, I_α and m denote the static moment of the whole airfoil around the elastic axis EO, the moment of inertia of the whole airfoil around the elastic axis EO and the mass of the whole profile, respectively. The coefficient S_β is the static moment of the flap of the airfoil around the flap axis EF and I_β is the moment of inertia of the flap of the airfoil around the flap axis EF. Constants $k_{hh}, k_{\alpha\alpha}, k_{\beta\beta}$ denote the spring stiffness of the flexible support of the airfoil and x_{1T} is the distance between the elastic axis EO and the flap axis EF. For simplification we shall use linearized equations in the matrix form

$$\widehat{\boldsymbol{K}}\boldsymbol{d}(t) + \widehat{\boldsymbol{B}}\dot{\boldsymbol{d}}(t) + \widehat{\boldsymbol{M}}\ddot{\boldsymbol{d}}(t) = \widehat{\boldsymbol{f}}(t), \tag{2.8}$$

where the stiffness matrix $\widehat{\boldsymbol{K}}$, the viscous damping matrix $\widehat{\boldsymbol{B}}$ and the mass matrix $\widehat{\boldsymbol{M}}$ have the form

$$\widehat{\boldsymbol{K}} = \begin{pmatrix} k_{hh} & 0 & 0 \\ 0 & k_{\alpha\alpha} & 0 \\ 0 & 0 & k_{\beta\beta} \end{pmatrix}, \quad \widehat{\boldsymbol{B}} = \begin{pmatrix} D_{hh} & 0 & 0 \\ 0 & D_{\alpha\alpha} & 0 \\ 0 & 0 & D_{\beta\beta} \end{pmatrix}, \tag{2.9}$$

$$\widehat{\boldsymbol{M}} = \begin{pmatrix} m & S_\alpha & S_\beta \\ S_\alpha & I_\alpha & I_\beta + x_{1T}S_\beta \\ S_\beta & I_\beta + x_{1T}S_\beta & I_\beta \end{pmatrix} \tag{2.10}$$

and the force vector $\widehat{\boldsymbol{f}}$ and the vector of the generalized coordinates $\boldsymbol{d}$ are given by

$$\widehat{\boldsymbol{f}}(t) = \begin{pmatrix} \mathcal{L}(t) \\ \mathcal{M}_\alpha(t) \\ \mathcal{M}_\beta(t) \end{pmatrix}, \quad \boldsymbol{d} = \begin{pmatrix} h(t) \\ \alpha(t) \\ \beta(t) \end{pmatrix}. \tag{2.11}$$

The components of the force vector $\widehat{\boldsymbol{f}}$ are given by

$$\begin{aligned} \mathcal{L} &= -\int_{\Gamma_{Wt}} \sum_{j=1}^{2} T_{2j} n_j \, \mathrm{d}S, \\ \mathcal{M}_\alpha &= -\int_{\Gamma_{Wt}} \sum_{i,j=1}^{2} T_{ij} n_j (-1)^i (x_{1+\delta_{1i}} - x_{1+\delta_{1i}}^{EO}) \, \mathrm{d}S, \\ \mathcal{M}_\beta &= -\int_{\Gamma_{Wt} \cap \partial K} \sum_{i,j=1}^{2} T_{ij} n_j (-1)^i (x_{1+\delta_{1i}} - x_{1+\delta_{1i}}^{EF}) \, \mathrm{d}S, \end{aligned} \tag{2.12}$$

where $\boldsymbol{n} = (n_1, n_2)$ is the outer unit normal to $\Gamma_{Wt} \subset \partial\Omega_t$. The stress tensor T_{ij} is computed from the velocity and pressure fields of the flow:

$$T_{ij} = \varrho \left[-p\delta_{ij} + \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]. \tag{2.13}$$

2.3. Initial and boundary conditions. The Navier-Stokes equations are completed by the initial condition

$$\boldsymbol{u}(\boldsymbol{x}, 0) = \boldsymbol{u}_0, \quad \boldsymbol{x} \in \Omega_0, \tag{2.14}$$

and the following boundary conditions. On Γ_D we prescribe the Dirichlet condition

$$\boldsymbol{u}|_{\Gamma_D} = \boldsymbol{u}_D. \tag{2.15}$$

On the outlet Γ_O we consider the so-called do-nothing boundary condition

$$-(p - p_{ref})\, \boldsymbol{n} + \nu \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}} = 0 \quad \text{on } \Gamma_O, \tag{2.16}$$

where p_{ref} is a given reference pressure and $\boldsymbol{n}$ denotes the unit outer normal to $\partial\Omega_t$. On Γ_{W_t} we consider the condition

$$\boldsymbol{u}|_{\Gamma_{W_t}} = \boldsymbol{w}|_{\Gamma_{W_t}}. \tag{2.17}$$

Moreover, we equip the system (2.8) with the initial conditions

$$\begin{aligned} \alpha(0) &= \alpha_0, & \dot{\alpha}(0) &= \alpha_1, \\ \beta(0) &= \beta_0, & \dot{\beta}(0) &= \beta_1, \\ h(0) &= h_0, & \dot{h}(0) &= h_1, \end{aligned} \tag{2.18}$$

where α_0, α_1, β_0, β_1, h_0, h_1 are input parameters of the model.

The initial value problem (2.8), (2.18) is transformed to a problem for a first-order system and then discretized by the fourth-order Runge-Kutta method.

The interaction of a fluid and an airfoil consists in the solution of the flow problem (2.6), (2.14) – (2.17) coupled with the structural model (2.8), (2.18). In what follows we shall be concerned with the discretization of the flow problem and describe the algorithm for the numerical solution of the complete fluid-structure interaction problem.

3. Discretization of the problem. We use equidistant partition of the time interval $[0, T]$, formed by $0 = t_0 < t_1 < \cdots < T$, $t_k = k\tau$, where $\tau > 0$ is a time step. On each time level we approximate the ALE derivative by the second-order backward difference formula and obtain the problem to find functions $\boldsymbol{u}^{n+1} : \Omega_{t_{n+1}} \mapsto I\!R^2$ and $p^{n+1} : \Omega_{t_{n+1}} \mapsto I\!R$ such that

$$\begin{aligned} \tfrac{3\boldsymbol{u}^{n+1} - 4\widehat{\boldsymbol{u}}^{n} + \widehat{\boldsymbol{u}}^{n-1}}{2\tau} + \big((\boldsymbol{u}^{n+1} - \boldsymbol{w}^{n+1}) \cdot \nabla\big)\, \boldsymbol{u}^{n+1} & \\ -\nu \Delta \boldsymbol{u}^{n+1} + \nabla p^{n+1} &= 0 \qquad \text{in } \Omega_{t_{n+1}}. \\ \operatorname{div} \boldsymbol{u}^{n+1} &= 0 \end{aligned} \tag{3.1}$$

This system is considered with the boundary conditions (2.15), (2.16), (2.17). The symbols $\hat{\boldsymbol{u}}^n$ and $\hat{\boldsymbol{u}}^{n-1}$ mean the functions $\boldsymbol{u}^n$ and $\boldsymbol{u}^{n-1}$ transformed from the domain Ω_{t_n} and $\Omega_{t_{n-1}}$ to the domain $\Omega_{t_{n+1}}$ using the ALE mapping: $\hat{\boldsymbol{u}}^i = \boldsymbol{u}^i \circ \boldsymbol{A}_{t_i} \circ \boldsymbol{A}_{t_{n+1}}^{-1}$, $i = n-1, n$

The starting point for the approximate solution in space is the weak formulation of problem (3.1). For this purpose we use the appropriate function spaces $W = (H^1(\Omega))^2$, $X = \{\mathbf{v} \in W; \mathbf{v}|_{\Gamma_D \cup \Gamma_{W_t}} = 0\}$ and $M = L^2(\Omega)$, where $t = t_{n+1}$ and $\Omega = \Omega_{t_{n+1}}$. Here $H^1(\Omega)$ denotes the Sobolev space and $L^2(\Omega)$ is the Lebesgue space of square integrable functions. We introduce the notation

$$\begin{aligned} a(U^*, U, V) = {} & \tfrac{3}{2\tau}(\boldsymbol{u}, \boldsymbol{v}) + \nu((\boldsymbol{u}, \boldsymbol{v})) + (((\boldsymbol{u}^* - \boldsymbol{w}^{n+1}) \cdot \nabla)\, \boldsymbol{u}, \boldsymbol{v}) \\ & -(p, \nabla \cdot \boldsymbol{v}) + (\nabla \cdot \boldsymbol{u}, q) \\ f(V) = {} & \tfrac{1}{2\tau} \Big(4\widehat{\boldsymbol{u}}^n - \widehat{\boldsymbol{u}}^{n-1}, \boldsymbol{v}\Big) - \textstyle\int_{\Gamma_O} \boldsymbol{v} \cdot \boldsymbol{n} \,\mathrm{d}S \end{aligned} \tag{3.2}$$

where

$$(a, b) = \int_\Omega ab \,\mathrm{d}x \tag{3.3}$$

and $U = (\boldsymbol{u}, p) \in W \times M, U^* = (\boldsymbol{u}^*, p) \in W \times M, V = (\boldsymbol{v}, q) \in X \times M$. The solution is $U = (\mathbf{u}, p)$ such that

$$U \in W \times M, \quad a(U, U, V) = f(V), \quad \forall V = (\mathbf{v}, q) \in X \times M, \tag{3.4}$$

and $\mathbf{u}$ satisfies the boundary conditions (2.15) and (2.17).

Now we define an approximate solution. We approximate the spaces W, X, M by their finite dimensional subspaces $W_\Delta, X_\Delta, M_\Delta$, $\Delta \in (0, \Delta_0)$, $\Delta_0 > 0$, where

$$X_\Delta = \left\{ \boldsymbol{v} \in W_\Delta;\, \boldsymbol{v}|_{\Gamma_D \cup \Gamma_{W_t}} = 0 \right\}. \tag{3.5}$$

This means that for each $\Delta \in (0, \Delta_0)$ we assign finite dimensional subspaces W_Δ, X_Δ, M_Δ, with dimensions $\dim W_\Delta = n_W(\Delta)$, $\dim X_\Delta = n_X(\Delta)$, $\dim M_\Delta = n_M(\Delta)$. The approximate solution is defined as a couple $U_\Delta = (\boldsymbol{u}_\Delta, p_\Delta) \in W_\Delta \times M_\Delta$ such that

$$a(U_\Delta, U_\Delta, V_\Delta) = f(V_\Delta), \quad \forall\, V_\Delta = (\boldsymbol{v}_\Delta, q_\Delta) \in X_\Delta \times M_\Delta \tag{3.6}$$

and $\boldsymbol{u}_\Delta$ satisfies a suitable approximation of the boundary conditions (2.15) and (2.17). The finite elements spaces X_Δ a M_Δ must satisfy the Babuška-Brezzi (BB) condition, which guarantees the stability of the used scheme. In practical computations we use the Taylor-Hood P^2/P^1 elements. This means that the velocity components are piecewise quadratic functions and the pressure is a piecewise linear function. These elements satisfy the BB condition.

3.1. Stabilization of the finite element method. For high Reynolds numbers approximate solutions can contain nonphysical spurious oscillations. In order to avoid them, we shall apply the streamline upwind Petrov-Galerkin (SUPG) method together with div-div stabilization based on the forms

$$\begin{aligned} L_\Delta(U^*, U, V) &= \textstyle\sum_{K \in \mathcal{T}_\Delta} \delta_K \left(\frac{3}{2\tau} \boldsymbol{u} - \nu \Delta \boldsymbol{u} + (\overline{\boldsymbol{w}} \cdot \nabla)\, \boldsymbol{u} + \nabla p,\, (\overline{\boldsymbol{w}} \cdot \nabla)\, \boldsymbol{v} \right)_K \\ F_\Delta(V) &= \textstyle\sum_{K \in \mathcal{T}_\Delta} \delta_K \left(\frac{1}{2\tau} (4 \widehat{\boldsymbol{u}}^n - \widehat{\boldsymbol{u}}^{n-1}),\, (\overline{\boldsymbol{w}} \cdot \nabla)\, \boldsymbol{v} \right)_K, \\ P_\Delta(U, V) &= \textstyle\sum_{K \in \mathcal{T}_\Delta} \tau_K (\nabla \cdot \boldsymbol{u},\, \nabla \cdot \boldsymbol{v})_K, \end{aligned} \tag{3.7}$$

where

$$U = (\boldsymbol{u}, p) \quad U^* = (\boldsymbol{u}^*, p) \quad V = (\boldsymbol{v}, q),$$

$\delta_K, \tau_K \geq 0$ are suitable parameters, $\overline{\boldsymbol{w}} = \boldsymbol{u}^* - \boldsymbol{w}^{n+1}$ is the transport velocity and $(\cdot, \cdot)_K$ is the scalar product in the space $L^2(K)$ or $[L^2(K)]^2$.

The solution of the stabilized discrete problem is $U_\Delta = (\boldsymbol{u}_\Delta, p_\Delta) \in W_\Delta \times M_\Delta$, such that the velocity $\boldsymbol{u}_\Delta$ satisfies the approximation of the boundary conditions (2.15) on Γ_D and (2.17) on $\Gamma_{W_{t_{n+1}}}$ and

$$\begin{aligned} a_\Delta(U_\Delta, U_\Delta, V_\Delta) + L_\Delta(U_\Delta, U_\Delta, V_\Delta) + P_\Delta(U_\Delta, V_\Delta) &= f_\Delta(V_\Delta) + F_\Delta(V_\Delta), \\ \forall\, V_\Delta = (\boldsymbol{v}_\Delta, q_\Delta) &\in X_\Delta \times M_\Delta. \end{aligned} \tag{3.8}$$

If we solve the problem (3.8), we obtain the approximate solution at time t_{n+1}, i.e. $\boldsymbol{u}_h = \boldsymbol{u}_h^{n+1}$ and $p_h = p_h^{n+1}$ defined in the domain $\Omega = \Omega_{t_{n+1}}$.

The choice of parameters δ_K and τ_K follows the works [3] and [6]. The parameter δ_K is based on the transport velocity $\overline{\boldsymbol{w}} = \boldsymbol{u} - \boldsymbol{w}(t_{k+1})$ and the viscosity ν. We put

$$\delta_K = \delta^* \frac{h_K}{2 \|\overline{\boldsymbol{w}}\|_{L^\infty(K)}}\, \xi(\Re_K^{\overline{\boldsymbol{w}}}), \tag{3.9}$$

where

$$\Re_K^{\overline{\boldsymbol{w}}} = \frac{h_K \|\overline{\boldsymbol{w}}\|_{L^\infty(K)}}{2\nu} \tag{3.10}$$

is the so-called local Reynolds number and h_K is the size of the element K measured in the direction of $\overline{\boldsymbol{w}}$. The function $\xi(\cdot)$ is non-decreasing in dependence on $\Re_K^{\overline{\boldsymbol{w}}}$ in such a way, that for local convective dominance ($\Re_K^{\overline{\boldsymbol{w}}} > 1$) $\xi \to 1$ and for local diffusion dominance ($\Re_K^{\overline{\boldsymbol{w}}} < 1$) $\xi \to 0$. The function $\xi(\cdot)$ can be defined, e.g. by

$$\xi(\Re_K^{\overline{\boldsymbol{w}}}) = \min\left(\frac{\Re_K^{\overline{\boldsymbol{w}}}}{6}, 1\right). \tag{3.11}$$

The parameters τ_K are defined by

$$\tau_K = \tau^* h_K \max_{\overline{\Omega}} |\overline{\boldsymbol{w}}| \, \xi(Re_K^{\overline{\boldsymbol{w}}}), \quad \tau^* \in (0,1]. \tag{3.12}$$

In practical computations we use the values $\delta^* = 0.025$ and $\tau^* = 1$.

3.2. Treatment of the nonlinearity in the flow model. The nonlinear problem (3.8) is (on each time level) solved with the aid of the Oseen iterative process. Starting from an initial approximation $U_{\Delta,0}^{n+1}$ at time t_{n+1} and assuming that already iterate $U_{\Delta,m}^{n+1}$ has been computed, we define $U_{\Delta,m+1}^{n+1} \in W_\Delta \times M_\Delta$ by

$$\begin{aligned} a_\Delta(U_{\Delta,m}^{n+1}, U_{\Delta,m+1}^{n+1}, V_\Delta) + L_\Delta(U_{\Delta,m}^{n+1}, U_{\Delta,m+1}^{n+1}, V_\Delta) + P_\Delta(U_{\Delta,m+1}^{n+1}, V_\Delta) \\ = f_\Delta(V_\Delta) + F_\Delta(V_\Delta), \quad \forall V_\Delta = (\boldsymbol{v}_\Delta, q_\Delta) \in X_\Delta \times M_\Delta \end{aligned}. \tag{3.13}$$

We obtain a sequence $U_{\Delta,m}^{n+1}$, $m = 0, 1, \ldots$, which converges to the solution U_Δ^{n+1} of the equation (3.8). For each time level t_{n+1} we set $U_{\Delta,0}^{n+1} = (2\hat{\boldsymbol{u}}_\Delta^n - \hat{\boldsymbol{u}}_\Delta^{n-1}, p_\Delta^n)$. As numerical experiments show, only a few iterations (3.13) have to be computed on each time level. Problem (3.13) is equivalent to a linear algebraic system, which is solved by the direct solver UMFPACK ([1]), which works sufficiently fast for systems with up to 10^5 equations.

3.3. Construction of the ALE mapping. For the construction of the ALE mapping we employ the linear elasticity equations for small deformations

$$(\lambda + \mu)\nabla \operatorname{div} \boldsymbol{m} + \mu \Delta \boldsymbol{m} = 0 \quad \text{in}\, \Omega_0, \tag{3.14}$$

where λ and μ are the Lamé coefficients and the displacement $\boldsymbol{m}$ is defined in Ω_0. The boundary conditions for the displacement $\boldsymbol{m}$ is prescribed by $\boldsymbol{m}|_{\Gamma_D \cup \Gamma_O} = 0$ and $\boldsymbol{m}|_{\Gamma_{W_0}}$ is computed from the movement of the airfoil, which is given for each time t by the knowledge of the functions $h(t)$, $\alpha(t)$, $\beta(t)$. Solving equation (3.14) gives us the ALE mapping of the domain Ω_0 onto Ω_t by the relation

$$\boldsymbol{X} \mapsto \boldsymbol{x}(\boldsymbol{X}, t) = \boldsymbol{A}_t(\boldsymbol{X}) = \boldsymbol{X} + \boldsymbol{m} \tag{3.15}$$

for each time t. From the displacement $\boldsymbol{m}$ we construct the domain velocity $\boldsymbol{w}$ with the aid of the backward difference formula.

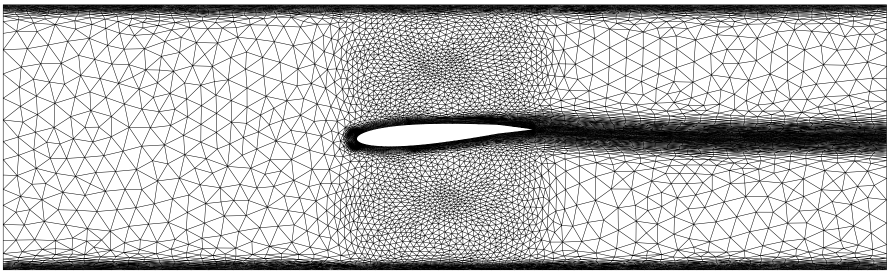

FIG. 4.1. *Anisotropically adapted mesh for NACA 0012 airfoil inserted into a wind tunnel*

4. Results. We performed computations for a number of configurations. Here we present results obtained for the airfoil NACA 0012 0.3 m long. The axis EO is placed at one third of the length of the airfoil measured from the leading edge and the axis EF is placed at 80 % of the length. The numerical simulation was carried out for the following data:

$$\begin{aligned} m &= 0.086622\,kg, & k_{hh} &= 105.109\,N/m, \\ k_{\alpha\alpha} &= 3.69558\,Nrad/m, & k_{\beta\beta} &= 0.2\,Nrad/m, \\ S_\alpha &= 0.000779598\,kgm, & S_\beta &= 0\,kgm, \\ I_\alpha &= 0.000487291\,kgm^2, & I_\beta &= 0.0000341104\,kgm^2, \\ x_{1T} &= 0.140001\,m, & D_{hh} &= 0\,Ns/m, \\ D_{\alpha\alpha} &= 0\,Ns\,rad/m, & D_{\beta\beta} &= 0\,Ns\,rad/m. \end{aligned}$$

For the computation we used an anisotropically adapted mesh, see Figure 4.1, which was obtained by the use of the software [2]. In Figure 4.2, the velocity isolines are shown for the inlet velocity 5 m/s for several time instants t marked in Figure 4.3 with graphs of the functions h, α, β in dependence on time.

REFERENCES

[1] T.A.DAVIS, *UMFPACK V4.0*, University of Florida.

[2] V.DOLEJŠÍ, *ANGENER V3.0*, Faculty of Mathematics and Physics, Charles University in Prague.

[3] T.GELHARD, G.LUBE, M.A.OLSHANSKII, J.H.STARCKE, *Stabilized finite element schemes with LBB-stable elements for incompressible flows*, J. Comput. Appl. Math. 177, 2005, pp.243–267.

[4] J.HORÁČEK, P.SVÁČEK, M.RŮŽIČKA, M.FEISTAUER, *Contribution to finite element modelling of airfoil aeroelastic instabilities*, Applied and Computational Mechanics 1, 2007, pp.43–52.

[5] V.LOSÍK AND, J.ČEČRDLE, *Výpočet flatru 3-dof modelu profilu křídla*, Výpočetní protokol P-PL-0061/07, Prague, 2007-320.

[6] G.LUBE, *Stabilized Galerkin finite element methods for convection dominated and incompressible flow problems*, Num. Anal. and Math. Model., Banach Center publications (29), Warszawa, 1994.

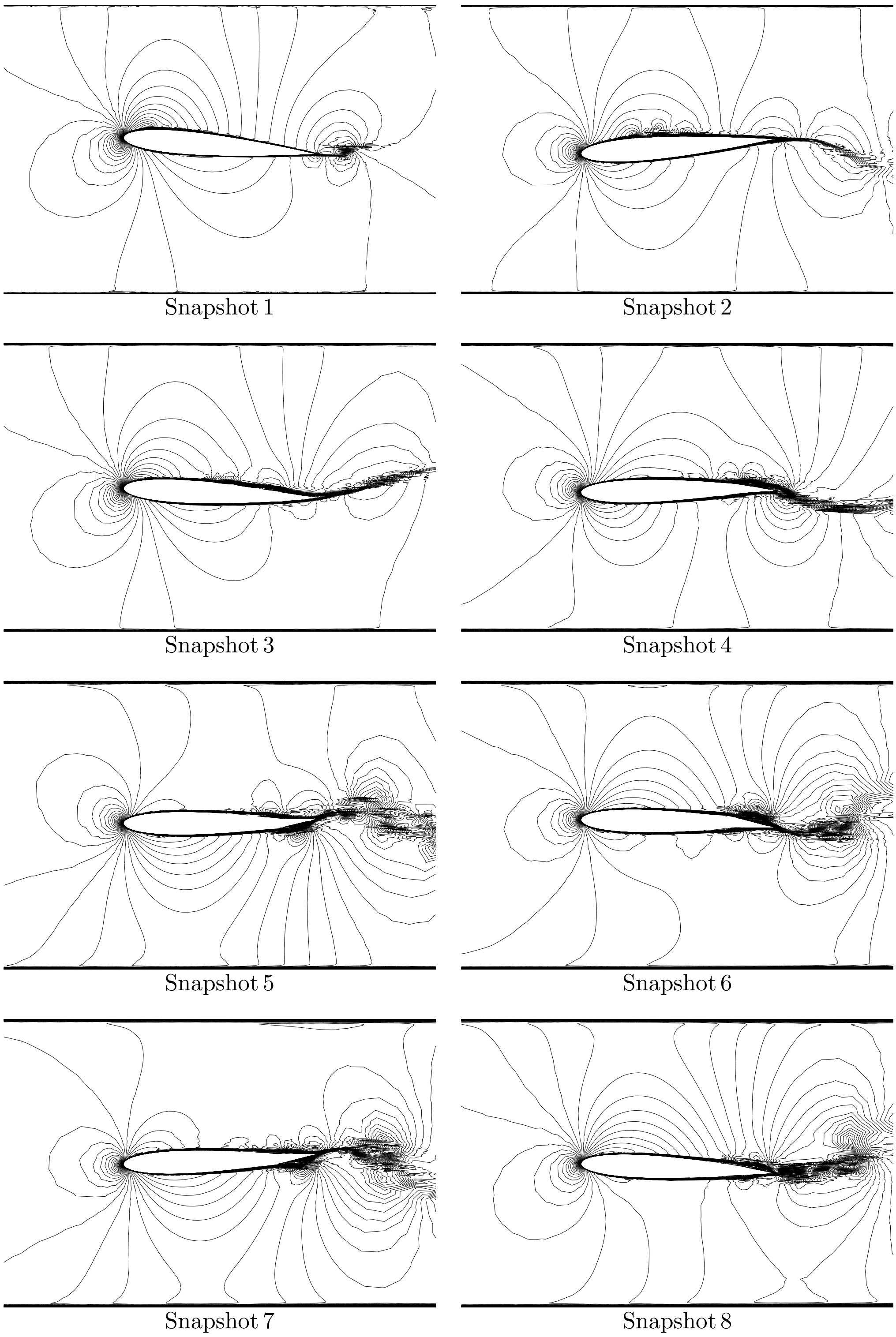

FIG. 4.2. *The velocity isolines for several time instants*

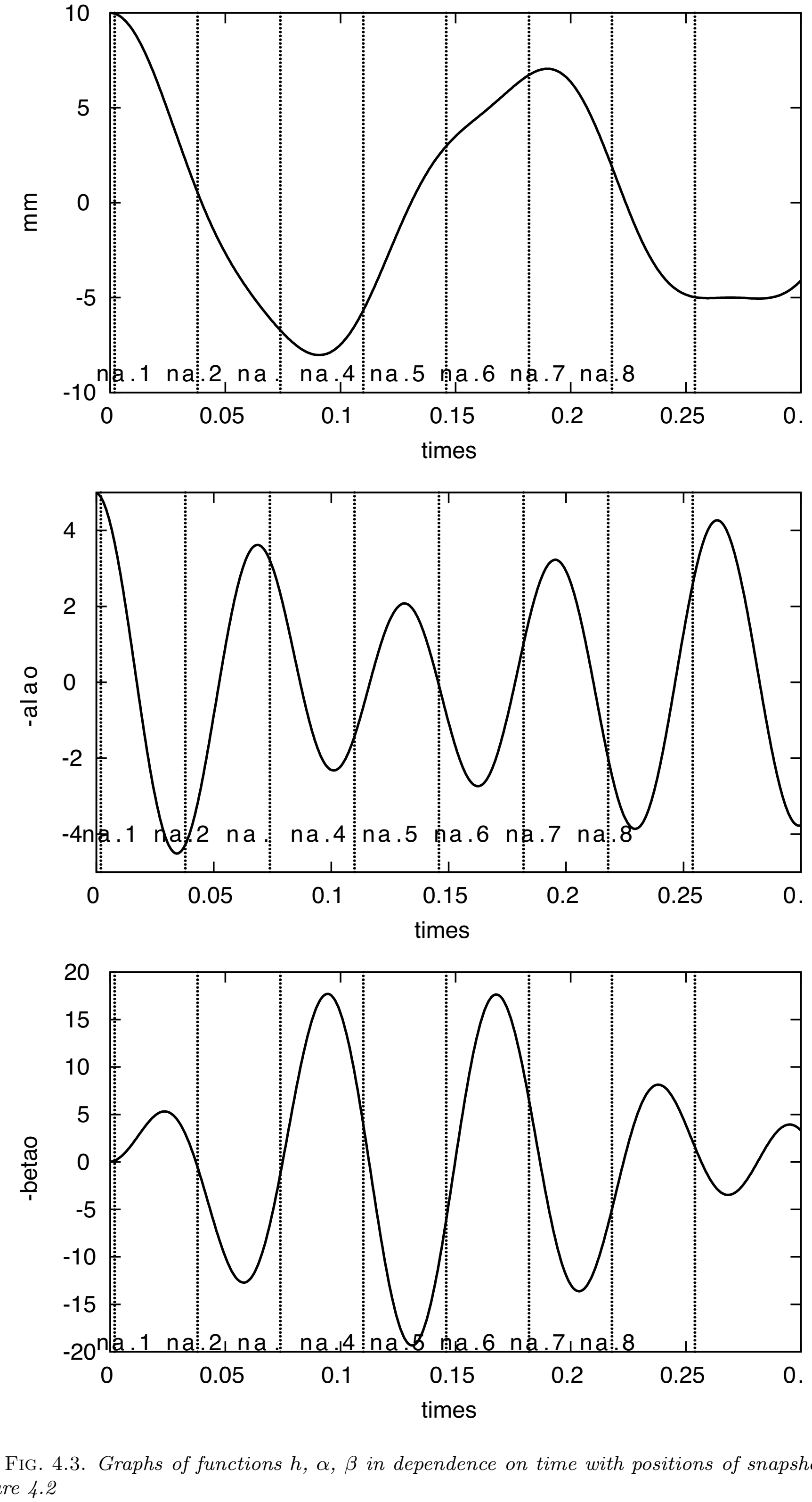

FIG. 4.3. *Graphs of functions h, α, β in dependence on time with positions of snapshots from Figure 4.2*