

# CERTAIN CLASSES OF $p$ -VALENT FUNCTIONS ASSOCIATED WITH WRIGHT'S GENERALIZED HYPERGEOMETRIC FUNCTIONS

G. MURUGUSUNDARAMOORTHY

ABSTRACT. The Wright's generalized hypergeometric function is used here to introduce a new class of  $p$ -valent functions  $\mathcal{WT}_p(\lambda, \alpha, \beta)$  defined in the open unit disc and investigate its various characteristics. Further we obtain distortion bounds, extreme points and radii of close-to-convexity, starlikeness and convexity of functions belonging to the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$ .

## 1. INTRODUCTION

Let  $\mathcal{A}(p)$  denote the class of functions of the form

$$(1.1) \quad f(z) = z^p + \sum_{n=k}^{\infty} a_n z^n, \quad p < k; \quad p, k \in \mathbb{N} = \{1, 2, 3, \dots\}$$

which are analytic in the open disc  $U = \{z : z \in \mathcal{C}; |z| < 1\}$ . For functions  $f \in \mathcal{A}(p)$  given by (1.1) and  $g \in \mathcal{A}(p)$  given by

$$g(z) = z^p + \sum_{n=k}^{\infty} b_n z^n, \quad p \in \mathbb{N} = \{1, 2, 3, \dots\}$$

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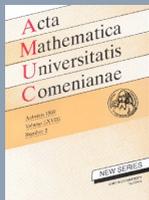


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we define the Hadamard product (or convolution) of  $f$  and  $g$  by

$$(1.2) \quad f(z) * g(z) = (f * g)(z) = z^p + \sum_{n=k}^{\infty} a_n b_n z^n, \quad z \in U.$$

For positive real parameters  $\alpha_1, A_1, \dots, \alpha_l, A_l$  and  $\beta_1, B_1, \dots, \beta_m, B_m$  ( $l, m \in \mathbb{N} = 1, 2, 3, \dots$ ) such that

$$1 + \sum_{n=k}^m B_n - \sum_{n=k}^l A_n \geq 0, \quad z \in U,$$

the Wright's generalized hypergeometric function [11]

$$\begin{aligned} {}_l\Psi_m[(\alpha_1, A_1), \dots, (\alpha_l, A_l); (\beta_1, B_1), \dots, (\beta_m, B_m); z] \\ = {}_l\Psi_m[(\alpha_j, A_j)_{1,l}(\beta_j, B_j)_{1,m}; z] \end{aligned}$$

is defined by

$$\begin{aligned} {}_l\Psi_m[(\alpha_j, A_j)_{1,l}(\beta_j, B_j)_{1,m}; z] \\ = \sum_{n=k}^{\infty} \left( \prod_{j=0}^l \Gamma(\alpha_j + nA_j) \right) \left( \prod_{j=0}^m \Gamma(\beta_j + nB_j) \right)^{-1} \frac{z^n}{n!}, \quad z \in U. \end{aligned}$$

If  $A_j = 1$  ( $j = 1, 2, \dots, l$ ) and  $B_j = 1$  ( $j = 1, 2, \dots, m$ ), we have the relationship:

$$(1.3) \quad \begin{aligned} {}_l\Psi_m[(\alpha_j, 1)_{1,l}(\beta_j, 1)_{1,m}; z] &\equiv {}_lF_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m; z) \\ &= \sum_{n=k}^{\infty} \frac{(\alpha_1)_n \dots (\alpha_l)_n}{(\beta_1)_n \dots (\beta_m)_n} \frac{z^n}{n!} \end{aligned}$$

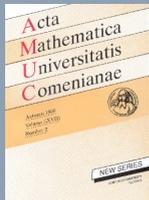


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( $l \leq m + 1$ ;  $l, m \in N_0 = N \cup \{0\}$ ;  $z \in U$ ) is the generalized hypergeometric function (see for details [2]) where  $(\alpha)_n$  is the Pochhammer symbol and

$$(1.4) \quad \Omega = \left( \prod_{j=0}^l \Gamma(\alpha_j) \right)^{-1} \left( \prod_{j=0}^m \Gamma(\beta_j) \right).$$

By using the generalized hypergeometric function Dziok and Srivastava [2] introduced the linear operator recently. In [3] Dziok and Raina extended the linear operator by using Wright's generalized hypergeometric function. First we define a function

$${}_l\phi_m[(\alpha_j, A_j)_{1,l}; (\beta_j, B_j)_{1,m}; z] = \Omega z^p {}_l\Psi_m[(\alpha_j, A_j)_{1,l}; (\beta_j, B_j)_{1,m}; z].$$

Let  $\Theta[(\alpha_j, A_j)_{1,l}; (\beta_j, B_j)_{1,m}] : \mathcal{A}(p) \rightarrow \mathcal{A}(p)$  be a linear operator defined by

$$\Theta[(\alpha_j, A_j)_{1,l}; (\beta_j, B_j)_{1,m}]f(z) := z^p {}_l\phi_m[(\alpha_j, A_j)_{1,l}; (\beta_j, B_j)_{1,m}; z] * f(z)$$

We observe that, for  $f(z)$  of the form (1.1), we have

$$(1.5) \quad \Theta[(\alpha_j, A_j)_{1,l}; (\beta_j, B_j)_{1,m}]f(z) = z^p + \sum_{n=k}^{\infty} \sigma_n a_n z^n$$

where  $\sigma_n$  is defined by

$$(1.6) \quad \sigma_n = \frac{\Omega \Gamma(\alpha_1 + A_1(n-p)) \dots \Gamma(\alpha_l + A_l(n-p))}{(n-p)! \Gamma(\beta_1 + B_1(n-p)) \dots \Gamma(\beta_m + B_m(n-p))}.$$

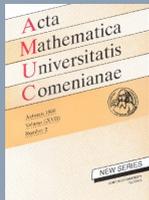


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For convenience, we write

$$(1.7) \quad \Theta[\alpha_1]f(z) = \Theta[(\alpha_1, A_1), \dots, (\alpha_l, A_l); (\beta_1, B_1), \dots, (\beta_m, B_m)]f(z)$$

Indeed, by setting  $A_j = 1 (j = 1, \dots, l)$ ,  $B_j = 1 (j = 1, \dots, m)$  and  $p = 1$  the linear operator  $\Theta[\alpha_1]$ , leads immediately to the Dziok-Srivastava operator [2] which contains, as its further special cases, such other linear operators of Geometric Function Theory as the Hohlov operator, the Carlson-Shaffer operator [1], the Ruscheweyh derivative operator [6], the generalized Bernardi-Libera-Livingston operator, the fractional derivative operator [8]. See also [2] and [3] in which comprehensive details of various other operators are given.

Motivated by the earlier works of [2, 4, 5, 7, 9, 10] we introduce a new subclass of  $p$ -valent functions with negative coefficients and discuss some interesting properties of this generalized function class.

For  $0 \leq \lambda \leq 1$ ,  $0 \leq \alpha < 1$  and  $0 < \beta \leq 1$ , we let  $\mathcal{W}_p(\lambda, \alpha, \beta)$  be the subclass of  $\mathcal{A}(p)$  consisting of functions of the form (1.1) and satisfying the inequality

$$(1.8) \quad \left| \frac{\mathcal{J}_\lambda(z) - 1}{\mathcal{J}_\lambda(z) + (1 - 2\alpha)} \right| < \beta \quad (z \in U)$$

where

$$(1.9) \quad \mathcal{J}_\lambda(z) = (1 - \lambda) \frac{\Theta[\alpha_1]f(z)}{z^p} + \lambda \frac{(\Theta[\alpha_1]f(z))'}{pz^{p-1}},$$

$\Theta[\alpha_1]f(z)$  is given by (1.7). Further let  $\mathcal{WT}_p(\lambda, \alpha, \beta) = \mathcal{W}_p(\lambda, \alpha, \beta) \cap T(p)$ , where

$$(1.10) \quad T(p) := \left\{ f \in \mathcal{A}(p) : f(z) = z^p - \sum_{n=k}^{\infty} a_n z^n, \quad a_n \geq 0; \quad z \in U \right\}.$$

The purpose of the present paper is to investigate the coefficient estimates, extreme points, distortion theorems and the radii of convexity and starlikeness of the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$ .



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## 2. COEFFICIENT BOUNDS

In this section we obtain coefficient estimates and extreme points of the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$ .

**Theorem 2.1.** *Let the function  $f$  be defined by (1.10). Then  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$  if and only if*

$$(2.1) \quad \sum_{n=k}^{\infty} (p + n\lambda - p\lambda)(1 + \beta)\sigma_n a_n \leq 2p\beta(1 - \alpha).$$

*Proof.* Suppose  $f$  satisfies (2.1). Then for  $z \in U$  we have

$$\begin{aligned} & |\mathcal{J}_\lambda(z) - 1| - \beta |\mathcal{J}_\lambda(z) + (1 - 2\alpha)| \\ &= \left| -\sum_{n=k}^{\infty} \frac{(p + n\lambda - p\lambda)}{p} (1 + \beta)\sigma_n a_n z^{n-p} \right| \\ &\quad - \beta \left| 2(1 - \alpha) - \sum_{n=k}^{\infty} \frac{(p + n\lambda - p\lambda)}{p} \sigma_n a_n z^{n-p} \right| \\ &\leq \sum_{n=k}^{\infty} \frac{(p + n\lambda - p\lambda)}{p} \sigma_n a_n - 2\beta(1 - \alpha) + \sum_{n=k}^{\infty} \frac{(p + n\lambda - p\lambda)}{p} \beta \sigma_n a_n \\ &= \sum_{n=k}^{\infty} \frac{(p + n\lambda - p\lambda)}{p} [1 + \beta]\sigma_n a_n - 2\beta(1 - \alpha) \leq 0. \end{aligned}$$

Hence, by maximum modulus theorem and (1.8),  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$ . To prove the converse assume that

$$\left| \frac{\mathcal{J}_\lambda(z) - 1}{\mathcal{J}_\lambda(z) + (1 - 2\alpha)} \right| = \left| \frac{-\sum_{n=k}^{\infty} \frac{(p+n\lambda-p\lambda)}{p} \sigma_n a_n z^{n-p}}{2(1-\alpha) - \sum_{n=k}^{\infty} \frac{(p+n\lambda-p\lambda)}{p} \sigma_n a_n z^{n-p}} \right| \leq \beta, \quad z \in U.$$

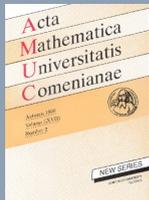


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Thus

$$(2.2) \quad \operatorname{Re} \left\{ \frac{\sum_{n=k}^{\infty} \frac{(p+n\lambda-p\lambda)}{p} a_n \sigma_n z^{n-p}}{2(1-\alpha) - \sum_{n=k}^{\infty} \frac{(p+n\lambda-p\lambda)}{p} \sigma_n a_n z^{n-p}} \right\} < \beta,$$

since  $\operatorname{Re}(z) \leq |z|$  for all  $z$ . Choose values of  $z$  on the real axis such that  $\mathcal{J}_\lambda(z)$  is real. Upon clearing the denominator in (2.2) and letting  $z \rightarrow 1^-$  through real values, we obtain the desired inequality (2.1).  $\square$

**Corollary 2.1.** *If  $f(z)$  of the form (1.10) is in  $\mathcal{WT}_p(\lambda, \alpha, \beta)$ , then*

$$(2.3) \quad a_n \leq \frac{2p\beta(1-\alpha)}{(p+n\lambda-p\lambda)[1+\beta]\sigma_n}, \quad n = k, k+1, \dots,$$

with the equality only for the function

$$(2.4) \quad f(z) = z^p - \frac{2p\beta(1-\alpha)}{(p+n\lambda-p\lambda)[1+\beta]\sigma_n} z^n, \quad n = k, k+1, \dots.$$

**Theorem 2.2** (Extreme Points). *Let*

$$(2.5) \quad \begin{aligned} f_p(z) &= z^p \quad \text{and} \\ f_n(z) &= z^p - \frac{2p\beta(1-\alpha)}{(p+n\lambda-p\lambda)[1+\beta]\sigma_n} z^n, \quad n = k, k+1, \dots \end{aligned}$$

Then  $f(z)$  is in the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$  if and only if it can be expressed in the form

$$(2.6) \quad f(z) = \mu_p z^p + \sum_{n=k}^{\infty} \mu_n f_n(z),$$

where  $\mu_n \geq 0$  and  $\mu_p + \sum_{n=k}^{\infty} \mu_n = 1$ .

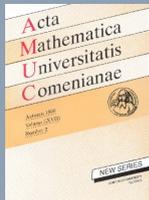


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*Proof.* Suppose  $f(z)$  can be written as in (2.6). Then

$$\begin{aligned} f(z) &= \mu_p z^p - \sum_{n=k}^{\infty} \mu_n \left[ z^p - \frac{2p\beta(1-\alpha)}{(p+n\lambda-p\lambda)[1+\beta]\sigma_n} z^n \right] \\ &= z^p - \sum_{n=k}^{\infty} \mu_n \frac{2p\beta(1-\alpha)}{(p+n\lambda-p\lambda)[1+\beta]\sigma_n} z^n. \end{aligned}$$

Now,

$$\begin{aligned} &\sum_{n=k}^{\infty} \frac{(p+n\lambda-p\lambda)[1+\beta]\sigma_n}{2p\beta(1-\alpha)} \mu_n \frac{2p\beta(1-\alpha)}{(p+n\lambda-p\lambda)[1+\beta]\sigma_n} \\ &= \sum_{n=k}^{\infty} \mu_n = 1 - \mu_p \leq 1. \end{aligned}$$

Thus  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$ . Conversely, let us have  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$ . Then by using (2.3), we set

$$\mu_n = \frac{(p+n\lambda-p\lambda)[1+\beta]\sigma_n a_n}{2p\beta(1-\alpha)}, \quad n \geq k$$

and  $\mu_p = 1 - \sum_{n=k}^{\infty} \mu_n$ . Then we have (2.6) and hence this completes the proof of Theorem 2.2.  $\square$

### 3. DISTORTION BOUNDS

In this section we obtain distortion bounds for the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$ .



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**Theorem 3.1.** Let  $f$  be in the class  $WT_p(\lambda, \alpha, \beta)$ ,  $|z| = r < 1$  and  $c_n = (p + n\lambda - p\lambda)\sigma_n$ . If the sequence  $\{c_k\}$  is nondecreasing for  $n > k$ , then

$$(3.1) \quad r^p - \frac{2p\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k} r^k \leq |f(z)| \leq r^p + \frac{2p\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k} r^k$$

$$(3.2) \quad pr^{p-1} - \frac{2pk\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k} r^{k-1} \leq |f'(z)| \leq pr^{p-1} + \frac{2pk\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k} r^{k-1}.$$

The bounds in (3.1) and (3.2) are sharp since the equalities are attained by the function

$$(3.3) \quad f(z) = z^p - \frac{2p\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k} z^k.$$

*Proof.* In the view of Theorem 2.1, we have

$$(3.4) \quad \sum_{n=k}^{\infty} a_n \leq \frac{2p\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k}$$

Using (1.10) and (3.4), we obtain

$$(3.5) \quad |z|^p - |z|^k \sum_{n=k}^{\infty} a_n \leq |f(z)| \leq |z|^p + |z|^k \sum_{n=k}^{\infty} a_n$$

$$r^p - r^k \frac{2p\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k} \leq |f(z)| \leq r^p + r^k \frac{2p\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k}.$$

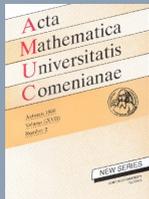


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Hence (3.1) follows from (3.5). Also,

$$|f'(z)| \leq pr^{p-1} + r^{k-1} \sum_{n=k}^{\infty} na_n \leq pr^{p-1} + r^{k-1} \frac{2pk\beta(1-\alpha)}{(p+k\lambda-p\lambda)[1+\beta]\sigma_k}.$$

Similarly, we can prove the left hand inequality given in (3.2) which completes the proof of the theorem.  $\square$

#### 4. RADIUS OF STARLIKENESS AND CONVEXITY

The radii of close-to-convexity, starlikeness and convexity for the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$  are given in this section.

**Theorem 4.1.** *Let the function  $f(z)$  defined by (1.10) belong to the class  $\mathcal{WT}_p(\lambda, \alpha, \beta)$ . Then  $f(z)$  is  $p$ -valently close-to-convex of order  $\delta$  ( $0 \leq \delta < p$ ) in the disc  $|z| < r_1$ , where*

$$(4.1) \quad r_1 := \inf_{n \geq k} \left[ \frac{(p-\delta)(p+n\lambda-p\lambda)[1+\beta]\sigma_n}{2pn\beta(1-\alpha)} \right]^{\frac{1}{n-p}}.$$

*Proof.* The function  $f \in T(p)$  is close-to-convex of order  $\delta$ , if

$$(4.2) \quad \left| \frac{f'(z)}{z^{p-1}} - p \right| < p - \delta.$$

For the left-hand side of (4.2) we have

$$\left| \frac{f'(z)}{z^{p-1}} - p \right| \leq \sum_{n=k}^{\infty} na_n |z|^{n-p}.$$

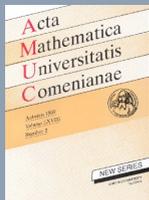


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The last expression is less than  $p - \delta$  if

$$\sum_{n=k}^{\infty} \frac{n}{p - \delta} a_n |z|^{n-p} < 1.$$

Using the fact that  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$  if and only if

$$\sum_{n=k}^{\infty} \frac{(p + n\lambda - p\lambda)[1 + \beta]\sigma_n a_n}{2p\beta(1 - \alpha)} \leq 1,$$

we can say (4.2) is true if

$$\frac{n}{p - \delta} |z|^{n-p} \leq \frac{(p + n\lambda - p\lambda)[1 + \beta]\sigma_n}{2p\beta(1 - \alpha)}.$$

Or, equivalently,

$$|z|^{n-p} = \left[ \frac{(p - \delta)(p + n\lambda - p\lambda)[1 + \beta] \sigma_n}{2pn\beta(1 - \alpha)} \right]$$

which completes the proof. □

**Theorem 4.2.** *Let  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$ . Then*

- (1)  *$f$  is  $p$ -valently starlike of order  $\delta$  ( $0 \leq \delta < p$ ) in the disc  $|z| < r_2$ ; that is,  $\operatorname{Re} \left\{ \frac{zf'(z)}{f(z)} \right\} > \delta$ , ( $|z| < r_2$ ) where*

$$r_2 = \inf_{n \geq k} \left\{ \frac{(p - \delta)(p + n\lambda - p\lambda)[1 + \beta] \sigma_n}{2p\beta(1 - \alpha)(k + p - \delta)} \right\}^{\frac{1}{n}}.$$

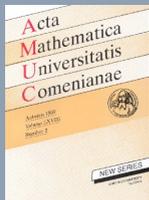


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(2)  $f$  is  $p$ -valently convex of order  $\delta$  ( $0 \leq \delta < p$ ) in the disc  $|z| < r_3$ , that is  $\operatorname{Re} \left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} > \delta$ , ( $|z| < r_3$ ) where

$$r_3 = \inf_{n \geq p+1} \left\{ \frac{(p-\delta)(p+n\lambda-p\lambda)[1+\beta]\sigma_n}{2n\beta(1-\alpha)(n-\delta)} \right\}^{\frac{1}{n}}.$$

*Proof.* (1) The function  $f \in T(p)$  is  $p$ -valently starlike of order  $\delta$ , if

$$(4.3) \quad \left| \frac{zf'(z)}{f(z)} - p \right| < p - \delta.$$

For the left hand side of (4.3) we have

$$\left| \frac{zf'(z)}{f(z)} - p \right| \leq \frac{\sum_{n=k}^{\infty} (n-p)a_n |z|^n}{1 - \sum_{n=k}^{\infty} a_n |z|^n}.$$

The last expression is less than  $p - \delta$  if

$$\sum_{n=k}^{\infty} \frac{n-\delta}{p-\delta} a_n |z|^n < 1.$$

Using the fact that  $f \in \mathcal{WT}_p(\lambda, \alpha, \beta)$  if and only if

$$\sum_{n=k}^{\infty} \frac{(p+n\lambda-p\lambda)[1+\beta]\sigma_n a_n}{2p\beta(1-\alpha)} < 1,$$

we can say (4.3) is true if

$$\frac{n-\delta}{p-\delta} |z|^n < \frac{(p+n\lambda-p\lambda)[1+\beta]\sigma_n}{2p\beta(1-\alpha)}.$$



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Or, equivalently,

$$|z|^n < \frac{(p - \delta)(p + n\lambda - p\lambda)[1 + \beta]\sigma_n}{2p\beta(1 - \alpha)(n - \delta)}$$

which yields the starlikeness of the family.

(2) Using the fact that  $f$  is convex if and only if  $zf'$  is starlike, we can prove (2), on lines similar to the proof of (1).  $\square$

*Remark.* In view of the relationship (1.3) the linear operator (1.5) and by setting  $A_j = 1$  ( $j = 1, \dots, l$ ) and  $B_j = 1$  ( $j = 1, \dots, m$ ) and specific choices of parameters  $l, m, \alpha_1, \beta_1$  the various results presented in this paper would provide interesting extensions and generalizations of  $p$ -valent function classes. The details involved in the derivations of such specializations of the results presented here are fairly straightforward.

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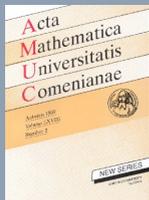
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G. Murugusundaramoorthy, School of Science and Humanities, VIT University, Vellore-632014, India,  
*e-mail*: [gms@vit.ac.in](mailto:gms@vit.ac.in)



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