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SEMILINEAR PROBLEMS PERTURBED THROUGH THE BOUNDARY CONDITION*

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Abstract. We analyze the semilinear diffusion equation $\Delta u = a(x)u^p$ in a smooth bounded domain Ω subjected to the boundary condition $\partial u/\partial\nu = \lambda u$, where ν is the outward unit normal to $\partial\Omega$, λ is a real parameter. The coefficient $a(x)$ is a nonnegative weight function, which could even vanish in a whole smooth subdomain Ω_0 of Ω. We consider both cases $p > 1$ and $0 < p < 1$, and give a detailed description of the existence, uniqueness or multiplicity and asymptotic behavior of nonnegative solutions. As an additional special feature, the adherence of the portions $\Omega\cap\partial\Omega_0$, $\partial\Omega\cap\partial\Omega_0$ of the boundary of Ω_0 are allowed to meet each other in a smooth manifold.

Key words. Bifurcation, Steklov problem, boundary blow-up, perturbation of domains.

AMS subject classifications. 35J25, 35B40.

1. Introduction.

In this note we are giving a summary of the issues of existence and uniqueness – alternatively multiplicity – of positive solutions to the semilinear problem:

$$\begin{cases} \Delta u = a(x)u^p & x \in \Omega \\ \dfrac{\partial u}{\partial \nu} = \lambda u & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a $C^{2,\alpha}$ bounded domain, $a \in C^\alpha(\overline{\Omega})$, $a(x) \geq 0$.

The main features of problem (1.1) are: a) the presence of the bifurcation parameter λ in the boundary condition, b) the possibility that $a \geq 0$ can vanish in a whole subdomain Ω_0 of Ω and c) the two regimes of the problem: "regular" and "degenerate", corresponding to the ranges $p > 1$ and $0 < p < 1$ of the exponent p respectively. In particular, we are paying special attention to the study of the asymptotic profiles of the solutions when the parameter λ tends to critical values λ_c where bifurcation phenomena occur. In the degenerate case, the study of the regions where the solutions u vanish identically ("dead cores") will be an additional issue to deal with.

Problem (1.1) constitutes another example of the group of models where a "dissipative" ("absorption") mechanism is in competition with another one of "production" ("radiation"). The logistic problem under either of the three classical boundary conditions is the paradigmatic example of this setting (see [8], [2], [3], [1] for the discussion of phenomena which are related to the ones covered here).

In the rest of the note we will assume that the coefficient $a(x)$ verifies the following hypothesis:

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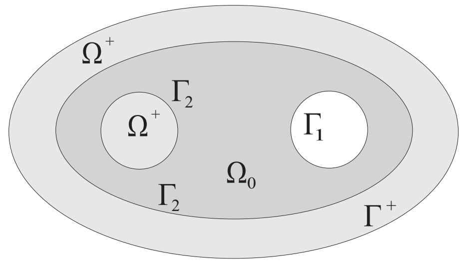

FIG. 1.1. *A possible configuration for an Ω_0 satisfying* H)$_s$.

H) *$a \in C^\alpha(\overline{\Omega})$ is nonnegative and nontrivial. Moreover, it is either strictly positive in Ω or $a \equiv 0$ in a subdomain $\Omega_0 \subset \Omega$ of class $C^{2,\alpha}$.*

For immediate reference we are fixing the following notations: $\Gamma_2 = \Omega \cap \partial\Omega_0$, $\Gamma_1 = \partial\Omega_0 \setminus \Gamma_2$. In addition, $\Omega^+ = \{x \in \Omega : a(x) > 0\}$, $\Gamma^+ = \partial\Omega^+ \cap \partial\Omega$. Notice that since $a(x)$ is nontrivial then both Γ_2 and Ω^+ must always be non void meanwhile $\Omega \cap \partial\Omega^+ = \Gamma_2$.

In our previous works [4], [5], an important part of the results was obtained under the simplifying assumption,

H)$_s$ $\overline{\Gamma}_2 = \Gamma_2$,

the bar standing for "adherence". This means that Γ_2 is a closed $N-1$ dimensional manifold and, more importantly, that Γ_1 is separated away from Γ_2 ($\text{dist}(\Gamma_1, \Gamma_2) > 0$). In particular, Ω^+ also defines a $C^{2,\alpha}$ subdomain of Ω, while $\Omega_0, \Omega^+, \Gamma_1, \Gamma_2, \Gamma^+$ consist only of a finite number of connected pieces (FIG.1.1). We are reviewing the above mentioned results in Sections 3 and 4.

In the present work some of the main statements in [4] are significantly extended to cover more natural configurations for Γ_1, Γ_2 in $\partial\Omega_0$ than in $H)_s$. In particular, we allow $\overline{\Gamma}_2 \cap \Gamma_1 \neq \emptyset$ (see details in Sections 3, 4 and Figure 3.2).

REMARK 1. The connectedness condition in Ω_0 can be relaxed without significant changes in the results.

2. Eigenvalue problems. In the analysis of (1.1) several eigenvalue problems which are non-standard appear. In fact, some of them involve the eigenvalue in the boundary condition while other ones exhibit a non usual regime for the coefficients.

A first eigenvalue problem is

$$\begin{cases} \Delta\phi = \lambda\phi & x \in \Omega_0 \\ \dfrac{\partial\phi}{\partial\nu} = \mu\phi & x \in \partial\Omega, \end{cases} \tag{2.1}$$

$\mu > 0$ regarded as a parameter. The existence of a principal eigenvalue $\lambda_1(\mu)$ to (2.1) and its qualitative behavior with respect to μ was first studied in [7] and extended in several aspects in [4]. The corresponding principal eigenfunctions ϕ are a source of sub and supersolutions for problem (1.1) in the regime $p > 1$.

The description of the set of positive solutions for problem (1.1) when $\Omega_0 \neq \emptyset$ requires the introduction of several critical values for λ, characterized by means of some eigenvalue problems. Specifically, we define $\lambda = \sigma_1$ as the principal eigenvalue of the mixed Steklov-

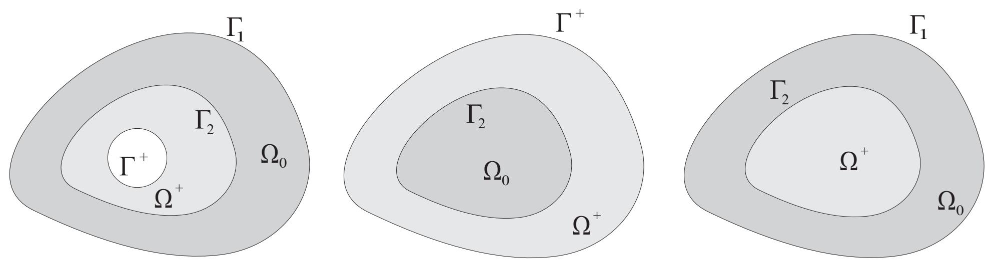

FIG. 2.1. *Several configurations for Ω_0 and Ω^+: Γ_1, $\Gamma^+ \neq \emptyset$, $\Gamma_1 = \emptyset$, $\Gamma^+ \neq \emptyset$ and $\Gamma_1 \neq \emptyset$, $\Gamma^+ = \emptyset$.*

type eigenvalue problem:

$$\begin{cases} \Delta\phi = 0 & x \in \Omega_0 \\ \phi = 0 & x \in \Gamma_2 \\ \dfrac{\partial\phi}{\partial\nu} = \lambda\phi & x \in \Gamma_1. \end{cases} \tag{2.2}$$

Since it could happen that $\Gamma_1 = \emptyset$, we set in that case $\sigma_1 := \infty$ (FIG. 2.1).

In the same way we introduce $\lambda = \sigma_1^+$, which is the corresponding principal eigenvalue of the problem:

$$\begin{cases} \Delta\phi = 0 & x \in \Omega^+ \\ \phi = 0 & x \in \Gamma_2 \\ \dfrac{\partial\phi}{\partial\nu} = \lambda\phi & x \in \Gamma^+. \end{cases} \tag{2.3}$$

We also set $\sigma_1^+ := \infty$ whenever $\Gamma^+ = \emptyset$.

We refer to [4] for a detailed study of the properties of the eigenvalue problems (2.2), (2.3) and some other related problems. We remark that σ_1^+ has to be suitably defined in case Ω^+ is not connected. In the first of the configurations in FIG. 2.1, both σ_1 and σ_1^+ are finite. One of them becomes infinite in the rest of the configurations.

3. The regime $p > 1$**.** We are next describing the main features of problem (1.1) for the regular case where $p > 1$. In THEOREMS 3.1 to 3.3 it is assumed that $a(x)$ satisfies H) together with the "separation" hypothesis $\mathrm{H})_s$. In the first statement the value $\sigma_1 = \infty$ (see Section 2) is also used when referring to the case $a(x) > 0$ in Ω. Such results are collected in [4]. The section concludes with the study of the case where Γ_1 and $\overline{\Gamma}_2$ meet in a nontrivial way.

THEOREM 3.1. *Assume $a(x)$ satisfies* H), $\mathrm{H})_s$ *while $p > 1$. Then problem* (1.1) *admits positive solutions $u \in C^{2,\alpha}(\overline{\Omega})$ if and only if:*

$$0 < \lambda < \sigma_1 \leq \infty,$$

the solution $u = u_\lambda(x)$ being unique in that case. Moreover, the mapping $\lambda \longmapsto u_\lambda$ is increasing and real analytic when considered with values in $C^{2,\alpha}(\overline{\Omega})$, and u_λ is globally attractive (among the positive solutions). Finally, $u_\lambda \to 0$ as $\lambda \to 0+$, while $|u_\lambda|_{\infty,\Omega} \to \infty$ when $\lambda \to \sigma_1$.

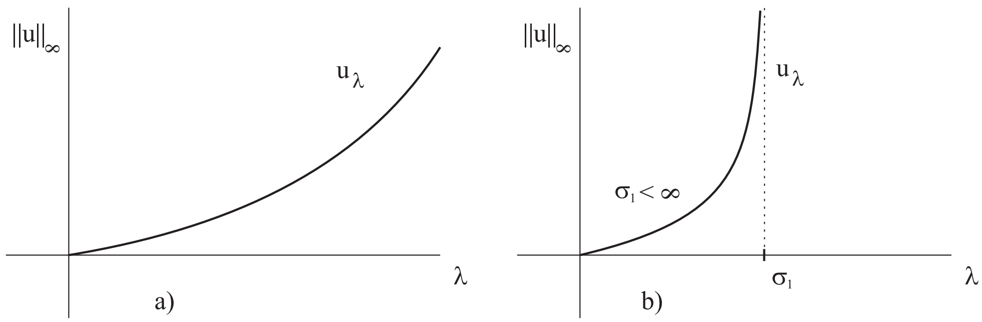

FIG. 3.1. *a) $a > 0$ in Ω or $\Omega_0 \neq \emptyset$ but $\sigma_1 = +\infty$, and b) $\sigma_1 < +\infty$.*

REMARK 2. THEOREM 3.1 allows the possibility that $a(x)$ vanishes somewhere on $\partial\Omega$ in the case where $a(x)$ is positive in Ω ($\Omega_0 = \emptyset$), also permitting the existence of zeros for $a(x)$ in Γ^+ if both Ω_0 and Γ^+ are nonempty ($a(x) \equiv 0$ on Γ_1!).

It has been established that if $\Omega_0 \neq \emptyset$ and $\sigma_1 < \infty$, positive solutions disappear when λ crosses the value σ_1. The next results explain this interruption by analyzing the singularities developed by the solutions. Even if $\sigma_1 = \infty$, the solution u_λ approaches a finite profile in Ω when λ tends to infinity.

THEOREM 3.2. *Assume $a(x) > 0$ for every $x \in \Omega$, or $\Omega_0 \neq \emptyset$ but $\sigma_1 = \infty$. Then the solution u_λ of* (1.1) *satisfies $u_\lambda \to u$ in $C^{2,\alpha}(\Omega)$ as $\lambda \to \infty$ where $u(x)$ is the minimal solution to the singular boundary value problem:*

$$\begin{cases} \Delta u = a(x)u^p & x \in \Omega \\ u = \infty & x \in \partial\Omega. \end{cases} \tag{3.1}$$

Moreover, we have the following estimate of the asymptotic growth rate:

$$\varliminf_{\lambda\to\infty} \lambda^{-\frac{2}{p-1}} \sup u_\lambda \geq |a|_\infty^{-\frac{1}{p-1}}.$$

THEOREM 3.3. *Under the condition $\sigma_1 < \infty$ and hence $\Gamma_1 = \partial\Omega_0 \cap \partial\Omega \neq \emptyset$, the solution u_λ to* (1.1) *satisfies $u_\lambda \to \infty$ uniformly in $\overline{\Omega}_0$ as $\lambda \to \sigma_1-$. In addition, $u_\lambda \to u$ in $C^{2,\alpha}(\Omega^+)$ where $u(x)$ is the minimal solution to the singular boundary value problem:*

$$\begin{cases} \Delta u = a(x)u^p & x \in \Omega^+ \\ u = \infty & x \in \Gamma_2 \\ \dfrac{\partial u}{\partial \nu} = \sigma_1 u & x \in \Gamma^+, \end{cases} \tag{3.2}$$

provided that $\Gamma^+ = \partial\Omega^+ \cap \partial\Omega \neq \emptyset$, or $u(x)$ is the minimal solution to the problem:

$$\begin{cases} \Delta u = a(x)u^p & x \in \Omega^+ \\ u = \infty & x \in \partial\Omega^+, \end{cases} \tag{3.3}$$

in case $\Gamma^+ = \emptyset$.

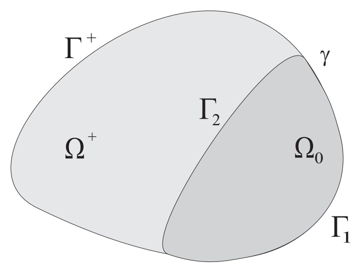

Fig. 3.2. *Configuration for Ω_0 with $\overline{\Gamma}_2$ and Γ_1 contacting in a smooth manifold γ.*

Remark 3. Observe that we prove the existence of nontrivial solutions of (3.2). This is a novelty in view of the boundary condition on Γ^+ not previously treated in the literature (see specially the corresponding problem in Theorem 3.4). On the other hand, suitable conditions on the weight $a(x)$ can be given in order to obtain uniqueness of positive solutions to the singular problems (3.1), (3.2) and (3.3).

We are next dealing with the hypothesis H) under a less restrictive assumption than $\mathrm{H})_s$ regarding the separation between Γ_1, $\overline{\Gamma}_2$. Namely, it will be assumed that $\Omega_0 \subset \Omega$ is a C^3 subdomain such that (recall that $\Gamma_2 = \Omega \cap \partial\Omega_0$, $\Gamma_1 = \partial\Omega \cap \partial\Omega_0$),

$\mathrm{H})_m$ $\Gamma_1, \overline{\Gamma}_2$ *are nonmepty $N-1$-dimensional manifolds having as common boundary γ, a $(N-2)$-dimensional closed submanifold of $\partial\Omega$* (Fig. 3.2).

The existence, under these conditions on Ω_0, of a unique principal weak eigenvalue $0 < \sigma_1 < \infty$ to (2.2) with a positive associated eigenfunction $\phi_1 \in H^1(\Omega_0) \cap W^{2,s}(\Omega_0)$, $1 < s < 4/3$, can be ensured by variational methods (see [5]).

Our main statement concerning this new framework for problem (1.1) essentially asserts that Theorems 3.1 and 3.3 are still valid even when Γ_1 and $\overline{\Gamma}_2$ meet each other.

Theorem 3.4. *Suppose that $a \in C^\alpha(\overline{\Omega})$ satisfies* H) *together with condition* $\mathrm{H})_m$. *Then problem* (1.1) *admits a weak positive solution in $H^1(\Omega)$ if and only if,*

$$0 < \lambda < \sigma_1.$$

Such solution is unique and defines indeed a classical solution in $C^{2,\alpha}(\overline{\Omega})$. As a mapping of $\lambda \in (0,\sigma_1)$, u_λ is increasing, smooth and bifurcates from zero at $\lambda = 0$. Moreover,

$$u_\lambda \to u,$$

as $\lambda \to \sigma_1-$ in $C^{2,\alpha}(\Omega^+) \cap C^{1,\alpha}(\Omega^+ \cup T)$ for every compact $T \subset \Gamma^+$ while,

$$u_\lambda \to \infty,$$

as $\lambda \to \sigma_1-$ uniformly on each compact set $K \subset \Gamma_2$. In particular:

$$\lim_{\mathrm{dist}(x,K) \to 0} u(x) = \infty,$$

and thus u defines a classical solution to problem (3.2),

$$\begin{cases} \Delta u = a(x)u^p & x \in \Omega^+ \\ u = \infty & x \in \Gamma_2 \\ \dfrac{\partial u}{\partial \nu} = \sigma_1 u & x \in \Gamma^+. \end{cases}$$

Remark 4. In contrast to the more regular case of the hypothesis $\mathrm{H})_s$, the approach to achieve existence in Theorem 3.4 is variational. In this regard, existence can be also obtained if Ω_0 is merely Lipschitz (cf. [4, Theorem 2]). An additional regularity argument then shows that any weak $H^1(\Omega)$ solution to (1.1) lies indeed in $L^\infty(\Omega)$ and hence its smoothness improves up to be a classical solution (see further details in [6]). On the other hand, it is a more subtle question to decide which is the possible asymptotic profile of u_λ in the interphase γ as $\lambda \to \sigma_1-$.

4. Degenerate regime. Let us now analyze problem (1.1) in the degenerate range $0 < p < 1$. The results we describe in what follows are contained in the work [5] with the exception of those involving the condition $\mathrm{H})_m$. We are denoting $\alpha_1 = \min\{p, \alpha\}$.

Theorem 4.1. *Assume that the coefficient $a \in C^\alpha(\overline{\Omega})$ satisfies $a(x) > 0$ for every $x \in \Omega$. Then:*

i) [Existence] *Problem (1.1) admits at least one nonnegative solution $u \in C^{2,\alpha_1}(\overline{\Omega})$, $u \neq 0$, for every $\lambda > 0$ and no nonnegative solutions exist if $\lambda \leq 0$.*

ii) [Bifurcation from infinity] *For certain $\lambda_0 > 0$ small enough and $0 < \lambda < \lambda_0$ there exists a unique positive solution $u_\lambda \in C^{2,\alpha}(\overline{\Omega})$. The mapping $\lambda \longmapsto u_\lambda$ is real analytic in $(0, \lambda_0)$ with values in $C^{2,\alpha}(\overline{\Omega})$. Moreover, it is decreasing and satisfies:*

$$\lim_{\lambda \to 0+} \lambda^{\frac{1}{1-p}}\, u_\lambda(x) = \left(\frac{1}{|\partial\Omega|} \int_\Omega a \right)^{\frac{1}{1-p}}$$

where the limit is taken in $C^{2,\alpha}(\overline{\Omega})$.

iii) [L^∞ estimate] *There exist constants $\lambda_1 > 0$, $C > 0$ such that every nonnegative solution u corresponding to $\lambda \geq \lambda_1$ satisfies:*

$$0 \leq u(x) \leq C\lambda^{-\frac{2}{1-p}}.$$

iv) [Dead core formation] *Every nonnegative solution $u_\lambda \neq 0$ corresponding to $\lambda \geq \lambda_2$, for a certain $\lambda_2 > 0$, develops a dead core $\mathcal{O}_\lambda = \{u_\lambda(x) = 0\}$ such that $\mathcal{O}_\lambda \to \Omega$ as $\lambda \to \infty$. More precisely:*

$$\{x : \operatorname{dist}(x, \partial\Omega) \geq d(\lambda)\} \subset \mathcal{O}_\lambda,$$

where $d(\lambda) \to 0+$ as $\lambda \to \infty$. Furthermore, $d(\lambda)$ can be chosen as $d(\lambda) = \dfrac{K}{\lambda}$, for a certain constant $K > 0$, provided that $a(x) > 0$ on $\partial\Omega$.

An important feature of the degenerate regime is that it exhibits multiple solutions when λ is sufficiently large. This is shown in the next result.

Theorem 4.2. *Let $\Omega \subset \mathbb{R}^N$ be a $C^{2,\alpha}$ domain whose boundary $\partial\Omega$ splits in k connected components, while $a \in C^\alpha(\overline{\Omega})$ is positive in Ω. Then problem (1.1) admits at least $2^k - 1$ nonnegative nontrivial solutions when λ is large enough.*

In view of this result, a question naturally arises: is the non connectedness of $\partial\Omega$ responsible for the multiplicity of solutions? The answer – in the negative – can be found in the next result, where Ω is a ball, hence $\partial\Omega$ is connected.

Theorem 4.3. *Consider problem (1.1) in a ball B of $\mathbb{R}^N$, with the coefficient $a(x)$ radial and positive. Then problem (1.1) admits for every $\lambda > 0$ a radial nonnegative solution $u \neq 0$. Such solution is unique for $0 < \lambda < \lambda_0$. Moreover:*

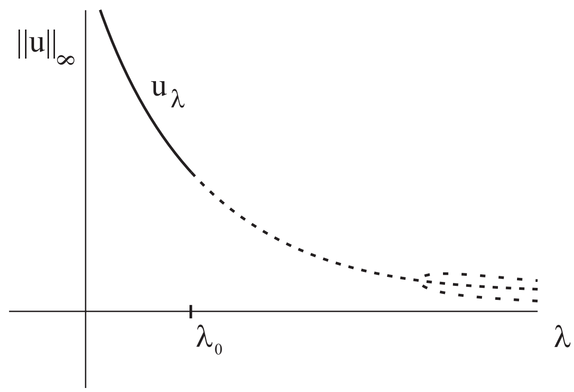

FIG. 4.1. *Bifuraction diagram for $a(x) > 0$: the continuous line means uniqueness.*

i) *There exists a* unique *radial nonnegative solution $u_\lambda \neq 0$ for large λ that satisfies:*

$$dist\ (\mathcal{O}_\lambda, \partial B) \sim \beta\lambda^{-1}, \qquad u_\lambda(1) \sim A\beta^\beta\lambda^{-\beta},$$

as $\lambda \to +\infty$ where $\beta = 2/(1-p)$, $A = [\beta(\beta-1)]^{-1/(1-p)}$ and $\mathcal{O}_\lambda = \{x \in B : u_\lambda = 0\}$.

ii) *There exists $\lambda_3 > 0$ such that problem* (1.1) *admits a solution $u \neq 0$, nonnegative and* nonradial *for every $\lambda \geq \lambda_3$.*

REMARK 5. Suppose that $a(x)$ satisfies H) with a nonempty domain Ω_0 so that $\sigma_1 = \infty$. Since this is equivalent to $\overline{\Omega}_0 \subset \Omega$ ($\Gamma_1 = \emptyset$, see Section 2) then $\mathrm{H})_s$ is satisfied and it is possible to deduce for problem (1.1) the same conclusions obtained in THEOREM 4.1. Namely:

i) For $\lambda > 0$, problem (1.1) admits at least a nonnegative nontrivial solution $u \in C^{2,\alpha_1}(\overline{\Omega})$, and no solutions exist if $\lambda \leq 0$.

ii) There exists a unique positive solution $u \in C^{2,\alpha}(\overline{\Omega})$ for $0 < \lambda < \lambda_0$ which bifurcates from infinity in $\lambda = 0$, with $u_\lambda(x) \sim \left(\frac{1}{|\partial\Omega|}\int_\Omega a\right)^{\frac{1}{1-p}} \lambda^{-\frac{1}{1-p}}$ as $\lambda \to 0+$.

iii) For large λ, nonnegative solutions u verify the estimate $0 \leq u(x) \leq C\lambda^{-\frac{2}{1-p}}$, where the constant C does not depend on u.

iv) Also for large λ, all nonnegative solutions u develop a dead core $\mathcal{O}_\lambda$ which satisfies $\{x : \mathrm{dist}(x, \partial\Omega) \geq d(\lambda)\} \subset \mathcal{O}_\lambda = \{u_\lambda(x) = 0\}$, $d(\lambda) \to 0$ as $\lambda \to \infty$.

However, the situation is completely different to the regular regime $p > 1$ when $\sigma_1 < \infty$ ($\Gamma_1 \neq \emptyset$). In strong contrast with the regular regime $p > 1$, the eigenvalue σ_1^+ plays an important rôle in the degenerate case $0 < p < 1$. The next statement gives precise information on problem (1.1) when $\sigma_1 < \infty$.

THEOREM 4.4. *Suppose that $a(x)$ satisfies* H) *with $\sigma_1 < \infty$ and assume either of the conditions* $\mathrm{H})_s$ *or* $\mathrm{H})_m$. *Then there exists at least one nonnegative nontrivial solution $u \in C^{2,\alpha_1}(\overline{\Omega})$, $\alpha_1 = \min\{p, \alpha\}$, with the same properties as the corresponding nonnegative solutions described in* THEOREM 4.1 *whenever $0 < \lambda < \sigma_1$. Moreover:*

i) *All nonnegative solutions u corresponding to $\lambda \geq \sigma_1$ satisfy $u \equiv 0$ in Ω_0.*

ii) *If $\sigma_1^+ = \infty$, there do not exist nonnegative nontrivial solutions for $\lambda \geq \sigma_1$.*

iii) *If $\sigma_1^+ < \infty$, nonnegative nontrivial solutions can only occur in the range $\lambda > \sigma_1^+$.*

In particular, such solutions cannot exist for λ verifying:

$$\sigma_1 \leq \lambda \leq \sigma_1^+,$$

assuming $\sigma_1 \leq \sigma_1^+$ (see FIG. 4.2*).*

iv) *Assume that condition* H$)_s$ *holds. Then, for $\sigma_1^+ < \infty$ certain $\lambda_1 > \sigma_1^+$ exists such that problem* (1.1) *admits at least one nonnegative nontrivial solution u for every $\lambda \geq \lambda_1$. Such solutions satisfy the estimate:*

$$0 \leq u(x) \leq C\ \lambda^{-\frac{2}{1-p}},$$

and develop a dead core which verifies $\mathcal{O}_\lambda \to \Omega$ as $\lambda \to \infty$ in the form described in REMARK 5-iv).

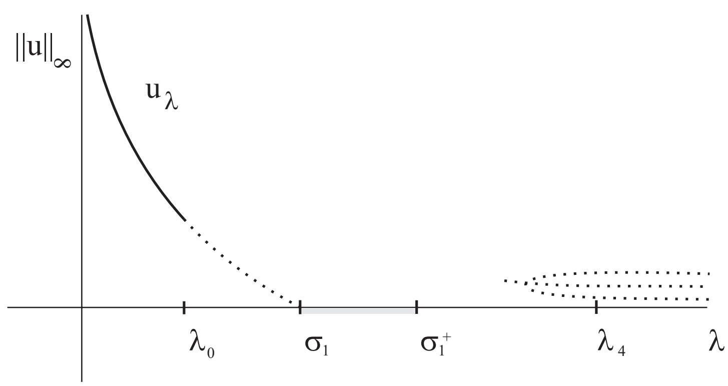

FIG. 4.2. *Bifurcation when $\sigma_1 < \infty$: the continuous line stands for uniqueness.*

To conclude with the description of the qualitative properties of nonnegative solutions to problem (1.1) in the degenerate case, we give sufficient conditions providing the existence of a bifurcation from $u = 0$ in $\lambda = \sigma_1$.

THEOREM 4.5. *Assume that $a(x)$ satisfies* H) *and either of the hypotheses* H$)_s$ *or* H$)_m$. *If $\sigma_1 < \infty$ and one of the following conditions holds: either $\sigma_1 \leq \sigma_1^+$ or $\sigma_1^+ < \sigma_1$ but $u = 0$ is the only nonnegative solution for $\lambda = \sigma_1$, then:*

$$u_\lambda \to 0 \qquad as\ \ \lambda \to \sigma_1-,$$

in $C^{2,\alpha_1}(\overline{\Omega})$. On the other hand, every possible solution develops a dead core $\mathcal{O}_\lambda \subset \Omega^+$ such that $\mathcal{O}_\lambda \to \Omega^+$ as $\lambda \to \sigma_1-$.

REMARK 6. Last assertion in THEOREM 4.5 implies that solutions u_λ of (1.1) converging to zero as $\lambda \to \sigma_1-$ develop their dead cores $\mathcal{O}_\lambda$ into Ω^+, being strictly positive in Ω_0. This should be contrasted with the behavior of nonnegative solutions to the problem (if any) in the range $\lambda \geq \sigma_1$. In fact, the latter must be identically zero in Ω_0 (THEOREM 4.4, i)).

REFERENCES

[1] Y. Du and Q. Huang, *Blow-up solutions for a class of semilinear elliptic and parabolic equations*, SIAM J. Math. Anal. **31** (1999), 1–18.

[2] J. M. Fraile, P. Koch-Medina, J. López-Gómez and S. Merino, *Elliptic eigenvalue problems and unbounded continua of positive solutions of a semilinear elliptic equation*, J. Differ. Equations **127** (1996), 295–319.

[3] J. García-Melián, R. Gómez-Reñasco, J. López-Gómez and J. Sabina de Lis, *Point-wise growth and uniqueness of positive solutions for a class of sublinear elliptic problems where bifurcation from infinity occurs*, Arch. Rat. Mech. Anal. **145** (1998), 261–289.

[4] J. García-Melián, J. D. Rossi and J. Sabina de Lis, *A bifurcation problem governed by the boundary condition I*, to appear in Non Linear Differential Equations Appl., NoDEA.

[5] J. García-Melián, J. D. Rossi and J. Sabina de Lis, *A bifurcation problem governed by the boundary condition II*. Preprint, 2005.

[6] J. García-Melián, J. D. Rossi and J. Sabina de Lis, *Existence and uniqueness of positive solutions to elliptic problems with nonlinear boundary conditions*. Submitted for publication.

[7] A. A. Lacey, J. R. Ockendon and J. Sabina, *Multidimensional reaction diffusion equations with nonlinear boundary conditions*, SIAM J. Appl. Math. **58** (1998), 1622–1647.

[8] T. Ouyang, *On the positive solutions of semilinear equations* $\Delta u + \lambda u - hu^p = 0$ *on the compact manifolds*, Trans. Am. Math. Soc. **331** (1992), 503–527.