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WAVELET TRANSFORMS IN SIGNAL AND IMAGE RESOLUTION ENHANCEMENT

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Abstract. Changing the resolution of a signal or image allows both global and detailed views of specific one-dimensional or two-dimensional signal components. The paper initially presents the use of the discrete Fourier transform in signal and image resolution enhancement. The main part of the paper is devoted to signal resolution selection using Wavelet functions. All algorithms presented in the paper have been verified in the Matlab environment.

Key words. Wavelet transform, time-scale signal decomposition, discrete Fourier transform, resolution enhancement

AMS subject classifications. 65T50, 65T60, 37M10

1. Introduction. A fundamental problem encountered in the digital processing of both one-dimensional and two-dimensional signals is the selection of the signal resolution. This defines the sampling period in the case of time series or the pixel distance in the case of images. Signal and image resolution enhancement is therefore also a fundamental problem in signal analysis.

2. Discrete Fourier Transform in Signal Resolution Enhancement. The discrete Fourier transform provides a very efficient tool for the detection of the harmonic components of a signal $\{x(n)\}_{n=0}^{N-1}$. The signal is transformed into the frequency domain by the relation

$$X(k) = \sum_{n=0}^{N-1} x(n)\ e^{-j\ kn\, 2\pi/N} \tag{2.1}$$

for $f(k) = k/N$ and $k = 0, 1, \cdots, N/2-1$. The evolution of the spectral components in time can also be found using the short time Fourier transform, which evaluates the spectral components in a moving window. The window length affects the resolution, with longer windows achieving finer frequency resolution but coarser time resolution and vice versa for shorter windows.

A similar principle can be applied to the analysis of an image represented by values $g(n,m)$ for $n{=}0,\cdots,N{-}1, m{=}0,\cdots,M{-}1$ of matrix $[g(n,m)]_{N,M}$ and resulting in values

$$G(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g(n,m) e^{-j\ kn\ 2\pi/N} e^{-j\ lm\ 2\pi/M} \tag{2.2}$$

for $k = 0, 1, \cdots, N/2-1$, $l = 0, 1, \cdots, M/2-1$ and frequency components

$$f_1(k) = k/N, \qquad f_2(l) = l/M \tag{2.3}$$

The principle of signal and image resolution enhancement is presented in Fig. 2.1 for an image matrix $[g(n,m)]_{N,M}$ taking into account that a one-dimensional signal can be considered as a special case of an image having one column only. In the *decomposition stage* the discrete Fourier transform is applied to the original matrix column

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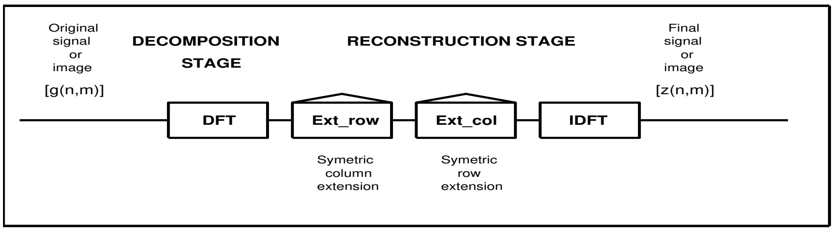

FIG. 2.1. *Principal of signal and image resolution enhancement by DFT*

by column at first. Denoting values of a selected column c of matrix $[g(n,m)]_{N,M}$ as $\{x(n)\}_{n=0}^{N-1} = \{g(n,c)\}_{n=0}^{N-1}$ it is possible to find their DFT $X(k)$ using Definition (2.1). The set of indices $k = -N/2, -N/2+1, \cdots, N/2-1$ imply normalized frequencies $f(k) = k/N \in \langle -0.5, 0.5)$ with a selected result presented in Fig. 2.2**(c)**. The inverse discrete Fourier transform (IDFT) applied to the sequence $X(k)$ result in the original sequence again. Signal enhancement can be achieved by symmetric extension of the original sequence $X(k)$ by zeros resulting in the sequence

(2.4) $[Z(-R/2),\cdots,Z(R/2-1)]^T = [0,\cdots,0,X(-N/2),\cdots,X(N/2-1),0,\cdots,0]^T$

for even values of $R > N$. The IDFT of sequence $Z(k)$ presented in Fig. 2.2**(d)** results in sequence

$$z(n) = \frac{1}{R}\sum_{k=-R/2}^{R/2-1} Z(k)\, e^{jkn\,2\pi/R} = \frac{1}{R}\sum_{k=-N/2}^{N/2-1} X(k)\, e^{jkn\,2\pi/R} \tag{2.5}$$

for $n = 0, 1, \cdots, R-1$. Evaluating for instance values of this sequence for $R = 2N$ and even indices only then

$$z(2n) = \frac{1}{2N}\sum_{k=-N/2}^{N/2-1} X(k)\, e^{j\,k\,2n\,2\pi/2N} \tag{2.6}$$

Comparing this result with the definition of the IDFT of sequence $X(k)$ in the form

$$x(n) = \frac{1}{N}\sum_{k=-N/2}^{N/2-1} X(k)\, e^{j\,kn\,2\pi/N} \tag{2.7}$$

it is obvious that in this case $x(n) = 2\,z(2n)$ and sequence $\{z(n)\}_{n=0}^{R-1}$ stands for interpolated sequence $\{x(n)\}_{n=0}^{N-1}$. Similar result can be derived for other values of $R > N$ and for odd value of N as well.

The *reconstruction stage* requires column extension by means of *ext_col* zero elements. In case of image processing, this is followed by the same process applied to

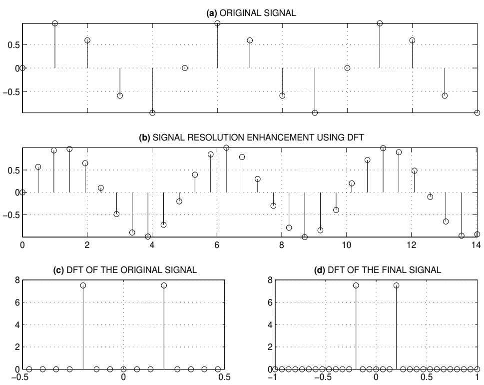

FIG. 2.2. *Signal DFT resolution enhancement*

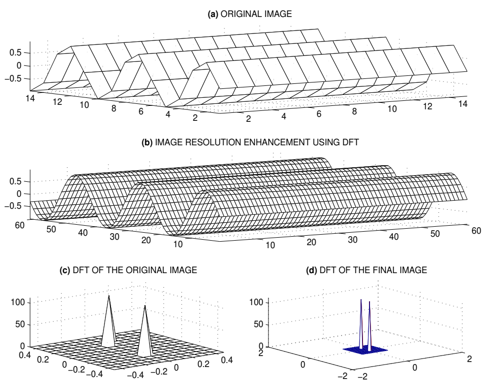

FIG. 2.3. *Image DFT resolution enhancement*

rows using factor *ext_row*. The inverse discrete Fourier transform is then applied to the extended signal or matrix. This whole process applied to signals or images allows

1. Decomposition and perfect reconstruction using $ext\_col = 0$ and $ext\_row = 0$
2. Resolution enhancement in case of $ext\_col \neq 0$ and $ext\_row \neq 0$

The process of signal resolution enhancement is presented in Fig. 2.2 for a simulated signal shown in Fig. 2.2**(a)**. After the evaluation of a given signal spectrum shown in Fig. 2.2**(c)** and its extension by zeros shown in Fig. 2.2**(d)** the initial signal can be reconstructed with its resolution enhanced as shown in Fig. 2.2**(b)**. A similar process applied to image resolution enhancement is presented in Fig. 2.3. The initial simulated image matrix shown in Fig. 2.3**(a)** is decomposed by the two dimensional discrete Fourier transform shown in Fig. 2.3**(c)** and its extension shown in Fig. 2.3**(d)**. Subsequent application of the inverse two-dimensional discrete Fourier transform results in the enhanced image matrix shown in Fig. 2.3**(b)**.

3. Principles of Discrete Wavelet Transform. Signal Wavelet decomposition using Wavelet transform (WT) provides an alternative to the discrete Fourier transform for signal analysis [7, 4] resulting in signal decomposition into two-dimensional functions of time and scale. The main benefit of WT over DFT is its multi-resolution time-scale analysis ability.

Wavelet functions used for signal analysis are derived from the initial function $W(t)$ forming basis for the set of functions

$$W_{m,k}(t) = \frac{1}{\sqrt{a}}\, W\left(\frac{1}{a}\,(t-b)\right) = \frac{1}{\sqrt{2^m}}\, W\left(2^{-m}t - k\right) \tag{3.1}$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k\,2^m$. Wavelet dilation, which is closely related to spectrum compression, enables local and global signal analysis.

4. Image Wavelet Decomposition and Reconstruction. The principle of signal and image decomposition and reconstruction for resolution enhancement is presented in Fig. 4.1 for an image matrix $[g(n,m)]_{N,M}$. Any one-dimensional signal can be considered as a special case of an image having one column only. *The decomposition stage* includes the processing the image matrix by columns at first using Wavelet (high-pass) and scaling (low-pass) function followed by row downsampling by factor D in stage $D.1$.

Let us denote a selected column of the image matrix $[g(n,m)]_{N,M}$ as signal $\{x(n)\}_{n=0}^{N-1} = [x(0), x(1), ..., x(N-1)]^T$. This signal can be analyzed by a half-band low-pass filter with its impulse response

$$\{s(n)\}_{n=0}^{L-1} = [s(0), s(1), \cdots, s(L-1)]^T \tag{4.1}$$

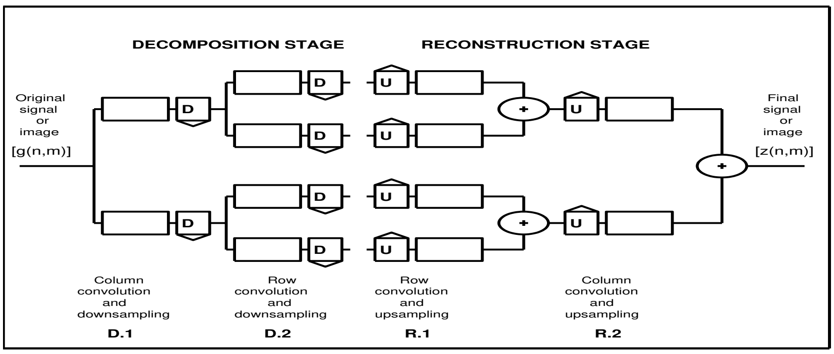

FIG. 4.1. *The principle of signal and image resolution enhancement by WT*

and corresponding high-pass filter based upon impulse response

$$\{w(n)\}_{n=0}^{L-1} = [w(0), w(1), \cdots, w(L-1)]^T \tag{4.2}$$

The first stage presented in Fig. 4.2 assumes the convolution of a given signal and the appropriate filter for decomposition at first by relations

$$xl(n) = \sum_{k=0}^{L-1} s(k)x(n-k) \qquad xh(n) = \sum_{k=0}^{L-1} w(k)x(n-k) \tag{4.3}$$

for all values of n followed by subsampling by factor D. In the following decomposition stage $D.2$ the same process is applied to rows of the image matrix followed by row downsampling. The decomposition stage results in this way in four images representing all combinations of low-pass and high-pass initial image matrix processing.

The *reconstruction stage* includes row upsampling by factor U at first and row convolution in stage $R.1$. The corresponding images are then summed. The final step $R.2$ assumes column upsampling and convolution with reconstruction filters followed by summation of the results again.

In the case of one-dimensional signal processing, steps $D.2$ and $R.1$ are omitted. The whole process can be used for

1. Signal/image decomposition and perfect reconstruction using D=2 and U=2
2. Signal/image resolution enhancement in the case of D=1 and U=2

The process of signal resolution enhancement is presented in Fig. 4.2 for a simulated signal shown in Fig. 4.2**(a)**. After the estimation of Wavelet coefficients shown in

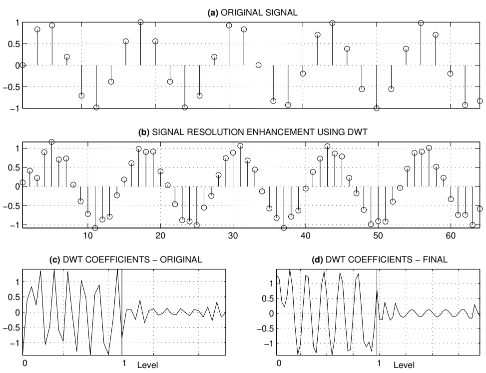

FIG. 4.2. *Signal WT resolution enhancement*

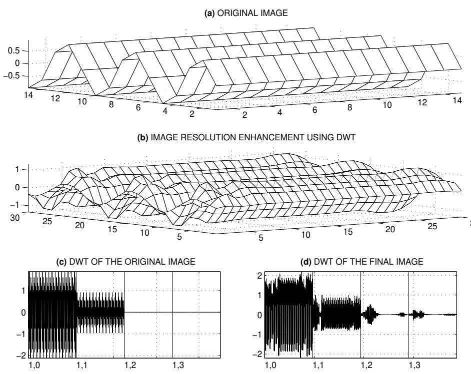

FIG. 4.3. *Image WT resolution enhancement*

Fig. 4.2**(c)** for $D=1$ the initial signal can be reconstructed with its resolution doubled as shown in Fig. 4.2**(b)**. A similar process applied to image resolution enhancement is presented in Fig. 4.3. The initial simulated image matrix shown in Fig. 4.3**(a)** is decomposed into Wavelet coefficients shown in Fig. 4.3**(c)** providing enhanced image matrix shown in Fig. 4.3**(b)**.

Algorithms for signal resolution enhancement using discrete Fourier transform and Wavelet transform are presented in Fig. 4.4 in the Matlab environment. Fundamental functions used in this program segment include the following functions:

$X = fft(x)$ – Fast Fourier transform of a given sequence $[x(0), x(1), ..., x(N-1)]^T$

$X = fftshift(X)$ – Shift of zero frequency component into the center of spectrum

$Y = wextend('1D', 'zpd', X, L)$ – Symetric extension of a given one dimensional signal by L zeros on both sides

$[c, l] = wevedec(x, level, wavelet)$ – Wavelet decomposition of the signal x at a specified *level* using selected *wavelet* providing vector c of approximation and detail wavelet coefficients and vector l of their lengthes

```
% Discrete Fourier Transform in Signal Resolution Enhancement
% N  - length of the sequence
% R  - new sequence length
% x  - given sequence
  X=fftshift(fft(x));
  Y=wextend('1D','zpd',X,(NZ-N)/2);
  y=R/N*ifft(Y);
% Wavelet Transform in Signal Resolution Enhancement
% wavelet - definition of Wavelet function
% l       - decomposition level
  [c,l]=wavedec(x,level,wavelet);
  [Lo_D,Hi_D,Lo_R,Hi_R]=wfilters(wavelet);
      XL=wconv('1D',x,Lo_D);    XH=wconv('1D',x,Hi_D);
      XL2=dyadup(XL);           XH2=dyadup(XH);
      XLL=wconv('1D',XL2,Lo_R); XHH=wconv('1D',XH2,Hi_R);
      z=XLL+XHH;
```

FIG. 4.4. *Algorithm of signal resolution enhancement by DFT and WT*

$[Lo\_D, Hi\_D, Lo\_R, Hi\_R] = wfilters(wavelet)$ – Definition of four filters associated with the orthogonal or biorthogonal *wavelet*

$XX = wconv('1D', x, f)$ – One dimensional convolution of a signal specified by vector $[x(0), x(1), ..., x(N-1)]^T$ and a selected filter f

$XX = dyadup(X)$ – Extended copy of vector X obtained by inserting zeros after each element of the given vector

A similar principle has been used for image resolution enhancement.

5. Applications of Image Resolution Enhancement. Signal and image resolution enhancement techniques are very important in digital signal processing. They have wide ranging applications in the analysis of time series and image processing, particularly in image compression, transmission and reconstruction. Special attention is paid to biomedical images, the classification of their structures and detection of their specific components. Both two-dimensional discrete Fourier transform and Wavelet transform provide efficient methods to solve these problems in the frequency and time domains.

Fig. 5.1 presents the application of the discrete Fourier transform to the resolution

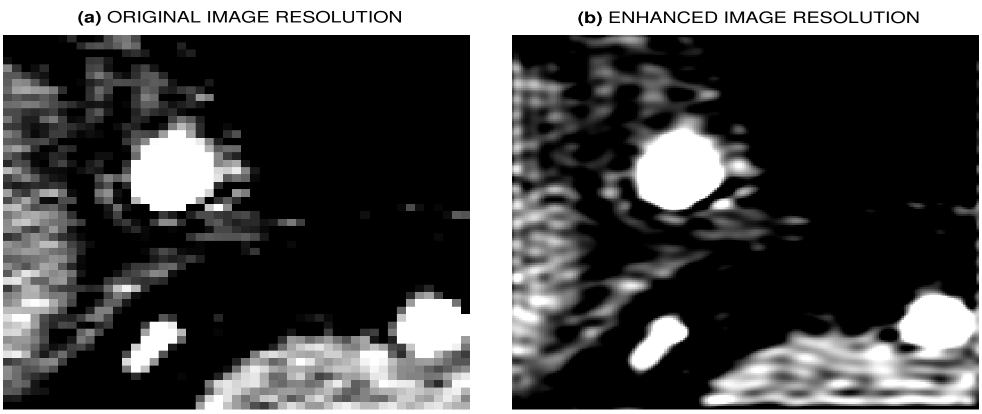

FIG. 5.1. *Application of the DFT for MR image resolution enhancement*

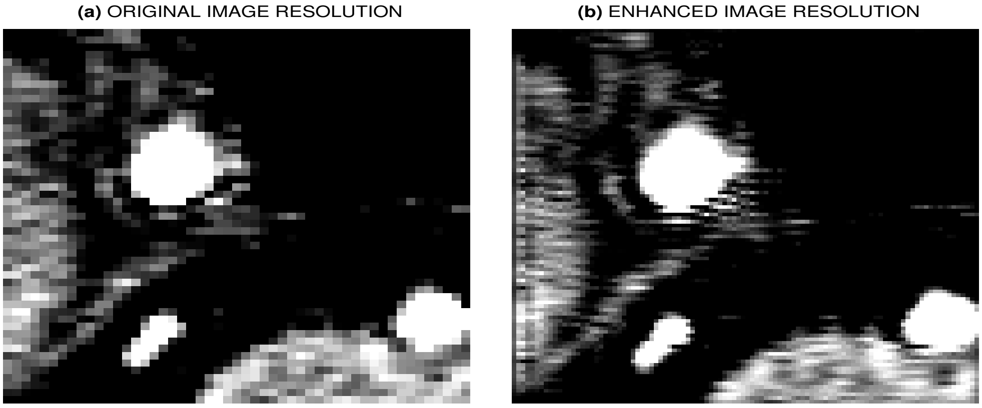

FIG. 5.2. *Application of the WT for MR image resolution enhancement*

enhancement of a magnetic resonance image of the brain. Similar results obtained by Wavelet transform are presented in Fig. 5.2. Comparison shows that, in each case, the image quality has been greatly enhanced, demonstrating the success of these methods. It is also possible to observe the problems resulting from periodic signal or image extension and boundary values estimation, especially in the case of the Wavelet transform application.

Table 5.1 presents selected results of using the Wavelet transform for image resolution enhancement with different Wavelet functions chosen from the given set available in the Wavelet transform library.

TABLE 5.1
MEAN SQUARE ERRORS (MSE) BETWEEN THE IMAGE OF MAGNETIC RESONANCE (MR) OF THE BRAIN ENHANCED BY THE DISCRETE FOURIER TRANSFORM AND SELECTED WAVELET FUNCTIONS

Method	*MSE*	*Method*	*MSE*
Haar Wavelet	0.3402	*Wavelet SYM2*	0.3677
Daubechies Wavelet DB3	0.5150	*Wavelet SYM4*	0.0976
Daubechies Wavelet DB4	0.7515	*Wavelet SYM8*	0.1147

6. Conclusion. Both in the case of DFT and WT it is possible to use various methods to enhance the resolution of one-dimensional and two-dimensional signals. The paper presents selected methods of the use of the Wavelet transform to achieve this goal. The results are compared with corresponding results obtained using the discrete Fourier transform. The algorithms used in the analysis have been verified initially with simulated signals and images and then applied to processing magnetic resonance images. Fig. 6.1 presents an example of such results corresponding to the algorithm presented above in Fig. 5.1.

It is assumed that further studies will be devoted to processing of both one-dimensional and two-dimensional signals. These studies will include prediction of one dimensional signals, analysis of boundary problems and de-noising, edge detection and classification of images.

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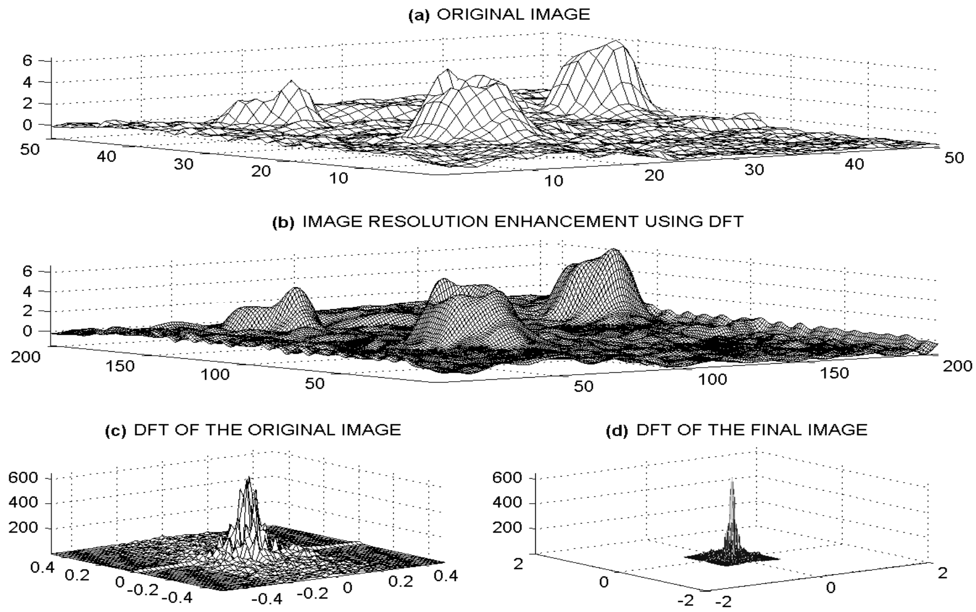

FIG. 6.1. *Application of the DFT for MR image resolution enhancement with results presented in three dimensions*

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