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DEA analysis for a large structured bank branch network

D. Ševčovič, M. Halická, P. Brunovský*

Institute of Applied Mathematics, Comenius University, 842 48 Bratislava, Slovak Republic (e-mail: sevcovic@fmph.uniba.sk)

Abstract In this paper we discuss results of Data Envelopment Analysis for the assessment of efficiency of a large structured network of bank branches. We focus on the problem of a suitable choice of efficiency measures and we show how these measures can influence results. As an underlying model we make use of the so-called normalized weighted additive model corresponding to variable returns to scale. Practical experiments were performed on large data sets provided by one of leading banks in Slovakia.

1 Introduction

A field of frequent and successful applications of Data Envelopment Analysis (DEA) is the evaluation of performance of bank branches. Applications of such a kind have been reported in many recent papers. An extensive source of information in this respect is the special issue of European Journal of Operational Research 98 (1997), in particular the paper Schaffnit, Rosen and Paradi (1997) and survey papers by Berger *et al* (1993,1997).

The present paper is focused on an application of DEA to an extensive structured network of bank branches. Theoretical as well as computational aspects of the application are presented. The methods developed for this application enable to assess the performance of units from different points of view. An important feature of the methods is that they admit small violation of the assumption of non-negativity of inputs or outputs.

The data were provided by the Slovenská Sporiteľňa (SLSP hereafter) - the largest Slovak bank which operates within the entire territory of Slovakia. Its orga-

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nizational structure consists of 37 regional *branch offices* located in major Slovak cities. Each of the branch offices runs various numbers (from 2 to 42) of smaller local organizational units called *subbranch offices* or *outlets*. The total amount of subbranch offices of SLSP is 591 (end of year 1998). Branch offices carry on wide range of banking operations. They can grant credits and they are in charge to invest money by means of various banking operations. Roughly speaking, branch offices are almost independent organizational subunits of SLSP. On the other hand, subbranch offices are responsible for basic banking services only. Normally, they can only carry on personal deposits and accounts and they are neither entitled to grant credits nor to perform other banking investments. Because of the principal qualitative differences in the ranges of activities of the two types of offices the analysis was performed on the set of 37 branches and on the set of 591 subbranches separately.

The paper is organized as follows. In the next section we discuss the analyzed data and we identify input and output factors characterizing branch activities satisfactorily. In Section 3 we present the DEA model we have chosen for our analysis. The model is described by a unit and translation invariant linear program in both primal and dual formulations. An important role in measuring performance of (sub) branches is played by the measure of efficiency. Various appropriate choices of this measure are discussed in Section 4. In Section 5 we present the results of the analysis. A special attention is put on comparison of results of different measures of efficiency. A correlation analysis of the results obtained by the methods is presented. In Section 6 we discuss the issues of model and measures selection as well as some computational aspects of the application.

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2 Structure and characterization of analyzed data

As it was already mentioned in Section 1, the variety of activities of branch offices is considerably richer than that of the subbranch offices. From the point of view of DEA it means that the former can be characterized by a higher number of inputs/outputs. Unfortunately, because of the low number of analyzed branches it was necessary to choose the number of significant inputs/outputs as small as possible. In fact, an undue large number of inputs/outputs relative to the number of DMU's makes most of them effective. Practical experience from extensive computations indicates that the total number of inputs and outputs should not be larger than one third of the number of units analyzed. For the analysis of branch offices we have considered 7 factors divided into 4 inputs (credits granted, banking expenditures, salaries and operational expenditures) and 3 outputs (credit profits, deposits, profit from banking operations). Their basic statistical properties are shown in Table 1.

The data extracted from the large network of subbranch offices have different properties. Unlike for branches, DEA analysis could have been performed on

Table 1 Mean value, standard deviation σ, minimal and maximal value of inputs and outputs for 37 major regional branch-offices.

Inputs (SKK)	Mean	σ	Min	Max
Credits granted	$1.52038\ 10^9$	$1.89381\ 10^9$	$3.40245\ 10^8$	$1.148\ 10^{10}$
Banking expendtrs	$3.62196\ 10^8$	$2.8084\ 10^8$	$9.74204\ 10^7$	$1.75\ 10^9$
Salaries	$3.80233\ 10^7$	$2.94929\ 10^7$	$1.04213\ 10^7$	$1.77506\ 10^8$
Oper. expendtrs	$4.28462\ 10^7$	$3.20938\ 10^7$	$1.75849\ 10^7$	$2.07432\ 10^8$
Outputs (SKK)				
Credit profits	$4.27221\ 10^8$	$7.45724\ 10^8$	$5.5309\ 10^7$	$4.64333\ 10^9$
Deposits	$2.55079\ 10^9$	$1.41585\ 10^9$	$6.90233\ 10^8$	$8.76865\ 10^9$
Banking profits	$4.95299\ 10^8$	$3.71137\ 10^8$	$9.86536\ 10^7$	$2.28804\ 10^9$

Table 2 Mean value, standard deviation σ, minimal and maximal value of inputs and outputs for 591 local branch-offices.

Inputs (SKK)	Mean	σ	Min	Max
Wage expendtrs	633814	$1.24914\ 10^6$	0	$9.72206\ 10^6$
Oper. expendtrs	796475	$1.50217\ 10^6$	40	$1.40686\ 10^7$
Except. expendtrs	259211	963460	0	$1.75314\ 10^7$
Outputs (SKK)				
Current accounts	$1.12648\ 10^7$	$3.01444\ 10^7$	$-1.432\ 10^6$	$2.92157\ 10^8$
Number of accounts (#)	797	1660	0	12396
Deposits	$8.46326\ 10^7$	$1.44295\ 10^8$	0	$1.17351\ 10^9$
Number of deposits (#)	3666	5906	0	43739

a larger number of input/output factors for subbranches because of a large number of the latter (591). However, the data of only 3 inputs (salaries, operational expenditures and exceptional other operational expenditures) and 4 outputs (current accounts and deposits and their corresponding numbers) were provided by all subbranches. Basic statistical properties of inputs/outputs of subbranches are presented in Table 2.

It is worth to note that the above mentioned choice of input/output quantities was based on particular requirements of the Slovak Saving Bank SLSP operating under conditions of transitional economy of Slovakia. For example, many of classified loans were moved into exceptional expenditures of branches.

The size of the subbranch offices differs widely. In such a case the choice of the type of returns to scale of the model appears to be crucial. Experience of the bank headquarters staff suggested that the subbranches of widely different sizes act in incomparable different conditions. This suggestion, confirmed by correlation analysis, lead us to choose the variable returns to scale model.

Finally, it is worth noting that the data provided by SLSP also contained non-positive numbers for some subbranches. A typical example are short credits as it can be seen from Table 2. This is why we were forced to choose translation invariant DEA models. Furthermore, a natural requirement for the DEA of bank branches is that the model should be unit (scale) invariant. This feature is of great importance in our analysis of SLSP because the ranges of inputs/outputs may differ by several orders of magnitude.

In summary, because of the above mentioned structure and qualitative properties of the given data sets of SLSP we had to choose a DEA variable returns to scale model which, in addition, is unit and translation invariant.

3 Model description

In this section we describe the DEA model used in our analysis of efficiency. The model, as a version of an additive weighted model, was first described by C.A. Knox Lovell and Jesus T. Pastor (1995) and called *normalized weighted additive model*. It corresponds to a variable returns to scale. In what follows, we briefly describe this model.

Consider a set of p decision making units (DMU's) each consuming given amounts of m inputs to produce n outputs. Let $x_j \in \mathbb{R}^m$ and $y_j \in \mathbb{R}^n$ denote the multidimensional vectors of inputs and outputs of the j-th DMU, $j = 1, ..., p$. By $o \in \{1, \ldots, p\}$ we denote the index of DMU to be analyzed. In order to evaluate DMU_o with input/output data vector (x_o, y_o) one may solve the normalized weighted additive DEA model, which is described by a linear program. We now present this model in both primal and dual formulations. The primal (dual) problem is frequently referred to as the envelopment (multiplier, respectively) form.

3.1 Primal normalized weighted additive model (P)

$$
\begin{aligned}
\min_{\lambda, s^+, s^-} \quad & -((w^-)^T s^- + (w^+)^T s^+) \\
s.t. \quad & \textstyle\sum_{j=1}^p x_j \lambda_j + s^- = x_o, && (1)\\
& \textstyle\sum_{j=1}^p y_j \lambda_j - s^+ = y_o, && (2)\\
& \textstyle\sum_{j=1}^p \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, ..., p, \\
& s^+ \geq 0, \quad s^- \geq 0. && (3)
\end{aligned}
$$

Here s^- and s^+ are m and n dimensional vectors of input and output slack variables. The m and n dimensional vectors w^- and w^+ are vectors of weights for input and output slack variables. They are defined by

$$w_i^- = (1/\sigma_i^-), \quad i = 1, ..., m \text{ and } w_i^+ = (1/\sigma_i^+), \quad i = 1, ..., n$$

where σ_i^- is the sample standard deviation of the i-th input variable and σ_i^+ the sample standard deviation of the i-th output variable. Note that DMU_o is rated as efficient if the optimal value to (P) is zero. In the other case it is inefficient.

3.2 Dual normalized weighted additive model (D)

$$\begin{aligned} \max_{u,v,z} \quad & v^T y_o - u^T x_o + z \\ s.t. \quad & v^T y_j - u^T x_j + z \leq 0, \quad j = 1, \dots, p \\ & u \geq w^-, \quad v \geq w^+, \quad z \in \mathbb{R} \end{aligned} \tag{4}$$

The variables u and v are the m and n dimensional vectors of local prices for inputs and outputs, respectively. The real variable z corresponds to the variable returns to scale equality constraint for the λ's in (P). Obviously, w^- and w^+ are the same as in the primal model. DMU_o is rated as efficient if and only if the optimal value of (D) is zero.

Let us remark that both (P) and (D) have an optimal solution. This follows from the fact that (P) as well as its dual (D) are feasible linear programs. Moreover, the objective functions (P) and (D) have a common optimal value to be denoted by F^*. Thus, it could seem that it does not matter whether (P) or (D) is being solved to evaluate the efficiency of a chosen DMU_o. However, not only the optimal value but also the optimal solution $(\lambda^*, s^{*+}, s^{*-})$ of (P) and the optimal solution (u^*, v^*, z^*) of (D) are important. Indeed, the primal optimal solution vector λ^* indicates a virtual unit belonging to the efficiency frontier with which the DMU_o is compared to. This virtual unit is described by the input vector x_f and the output vector y_f where

$$x_f := \sum_{j=1}^{p} \lambda_j^* x_j \qquad y_f := \sum_{j=1}^{p} \lambda_j^* y_j$$

and by (1) and (2) one has

$$x_f = x_o - s^{*-} \qquad y_f = y_o + s^{*+}. \tag{5}$$

The slack vectors s^{*+} and s^{*-} give measures of possible reserves in input and outputs, resp. when compared the actual unit (x_o, y_o) to the efficient virtual unit (x_f, y_f). On the other hand, the solution vectors u^* and v^* of the dual problem enable us to identify local prices of inputs and outputs for DMU_o. For example, they enter the formulas $u^{*T} x_o$ and $v^{*T} y_o$ for the virtual (one dimensional) input and output respectively, which are important in the concept of the so-called technical efficiency. Another interpretation of u^*, v^*, z^* and the objective function for the additive model is described e. g. in Chapter 2 of the book by Charnes at al. (1969). Finally, we recall that DMU_o is rated (by this model) as efficient if the optimal value F^* is zero. For inefficient DMU_o one has $F^* < 0$ and F^* can be interpreted as an inefficiency score. All this information gives a more complete and qualitative picture about every DMU_o and can be important for decision makers.

This model possesses the properties we are seeking for. It is well known that it is translation and unit invariant. Moreover, the results (the optimal value and the optimal solution) are not very sensitive on the data of the worst performing units. It is also easy to see that the efficiency score (given by the optimal value F^*) is monotone decreasing in each input and output slack. A disadvantage of this model is that the score is not *á-priori* bounded from below. This leads to difficulties when trying to transform this score into the bounded interval $[0, 1]$.

4 Definitions of three measures of efficiency

In this section we define three measures of efficiency which reflect the information obtained by solving the above described model in three different ways. The measures are normalized in such a way that their values for efficient units are equal to 1 and belong to the interval $[0, 1)$ for inefficient units.

4.1 Measure of efficiency based on optimal value

The first measure is obtained by a linear transformation of the optimal value F^* to the interval $[0, 1]$. Hence, we define the measure of efficiency E_O by

$$E_O := 1 + \varepsilon F^*$$

where $\varepsilon > 0$ is a scaling parameter to be chosen in such a way that $E_O \in [0, 1]$ for all analyzed units. Notice that one can choose $\varepsilon > 0$ in such a way that the lowest value E_O among all analyzed units is zero.

It turned out that by using this efficiency measure more than 95% of all subbranches of SLSP had their efficiencies between 0.9 and 1 (cf. Table 3, line E_O). Clearly, such a non-uniform distribution of efficiency is is not convenient from the point of view of decision making, since the accumulation of the 448 efficiency values into a very small interval makes the results badly readable and blind. Another objectionable property of this measure is that it is very sensitive on the choice of the parameter ε and hence on the efficiency score of the worst performing unit. Omitting of the three worst performing units from the set of subbranches of SLSP would dramatically change the efficiencies of all other subbranches. This is caused (in our case) by the great differences between the three worst performing units and the others units, and by the linear transformation used in definition of this measure.

Let us mention that one can overcome this difficulty by using of non-linear transformation. For example, Némethová (2001) proposed the choice: $\tilde{E}_O := e^{F^*}$. This non-linear approach can be generalized via the so-called contrast function $\phi : R_+ \to R$ satisfying

$$\phi(0) = 1, \quad \phi(r) > 0, \quad \phi'(r) < 0, \quad \lim_{r\to\infty} \phi(r) = 0\,.$$

Then, the efficiency measure can be defined as

$$\tilde{E}_O := \phi(-F^*).$$

As an example one can choose either $\phi(r) = e^{-\gamma r}$ or $\phi(r) = 1/(1 + \gamma r^2)$ where $\gamma > 0$ is a contrast parameter. However, in our simulation to follow we have chosen the first simplest form of the efficiency measure, i.e. $E_0 = 1 + \varepsilon F^*$.

4.2 Efficiency measure based on the primal model solution

As it was mentioned in Section 3, solving the primal problem one obtains the virtual efficient unit DMU_f with input/output vector (x_f, y_f) (see (5)). Using this information an efficient measure can be defined comparing DMU_o with DMU_f by means of the fractions of their particular inputs/outputs values. To derive such a measure we first introduce the following assumption

$$x_j > 0 \text{ and } y_j > 0 \text{ for } j = 1, ..., p. \tag{6}$$

Now, it is easy to see that (6) together with (3) gives $x_{fi} \leq x_{oi}$, $i = 1, \ldots, m$ and $y_{fi} \geq y_{oi}$, $i = 1, \ldots, n$. Therefore, the ratios

$$E_i^x := \frac{x_{fi}}{x_{oi}} = \frac{x_{oi} - s_i^{*-}}{x_{oi}}, \quad E_i^y := \frac{y_{oi}}{y_{fi}} = \frac{y_{oi}}{y_{oi} + s_i^{*+}}, \tag{7}$$

can be understood as partial fractional efficiencies of the corresponding inputs and outputs for DMU_o. Let us note that due to assumption (6) they are well defined, so that $E_i^x, E_i^y \in (0, 1]$. The aggregate efficiency measure can now be defined by several ways as weighted value of all partial efficiencies. In our case we have defined it as the mean value, i.e.

$$E_P := \sum_{i=1}^{m} \mu_i^- E_i^x + \sum_{i=1}^{n} \mu_i^+ E_i^y \tag{8}$$

where $\mu_i^{\pm}$ are positive weights such that $\sum_{i=1}^{m} \mu_i^- + \sum_{i=1}^{n} \mu_i^+ = 1$. In our simulations we chose the uniform weight distribution, i.e.

$$\mu_i^- = \mu_i^+ = \frac{1}{m+n}.$$

Another possible choice is based on weights attached to the slacks in the objective function in the primal model (P), i.e.

$$\mu_i^{\pm} = \frac{w_i^{\pm}}{\sum_{j=1}^{m} w_j^- + \sum_{j=1}^{n} w_j^+}.$$

Substituting (7) and (5) into (8) one obtains

$$E_P = \sum_{i=1}^{m} \mu_i^- \frac{x_{oi} - s_i^{*-}}{x_{oi}} + \sum_{i=1}^{n} \mu_i^+ \frac{y_{oi}}{y_{oi} + s_i^{*+}}.$$

It is easy to see that E_P is unit invariant but it is not translation invariant. In fact, multiplication of the i-th input or output of all DMUs by a positive constant does not change the value of E_P. However, adding some constant a to the i-th input (output) of all DMUs we change E_P. Indeed, the corresponding E_i^x (E_i^y) will increase or decrease depending on the plus or minus sign of a, respectively. Let us also remark that our data do not fulfill assumption (6). However, solving the primal problem for our data we observed that $x_{fi} \geq 0$ and $y_{fi} \geq 0$. In this case the definitions of partial efficiencies (7) entering (8) can be modified as follows: If $x_{oi} = 0$, then $E_i^x := 1$ and, if $y_{oi} \leq 0$, then $E_i^y := 0$. We will refer to this method as the primal method.

Recently, Cooper, Seiford and Tone (2000) investigated the so-called Slack-Based measure efficiency model (SBM). The measure of efficiency is defined as

$$E_S := \frac{\frac{1}{m}\sum_{i=1}^m E_i^x}{\frac{1}{n}\sum_{i=1}^n 1/E_i^y}.$$

Similarly as in our choice of measure E_P, the SBM model rates efficiencies E_i^x (E_i^y) of particular inputs (outputs) uniformly, i.e. their weights μ_i^-, μ_i^+ are equal. However, SBM model takes the measure E_S as an objective function and hence the results of optimization depend on the definition of this measure. It would be of interest to compare and test results obtained by our normalized weighted model with the efficiency measure E_P to those of SBM model.

4.3 Efficiency measure based on the dual model solution

Finally, we introduce an efficiency measure based on the virtual input and output mentioned in Section 3. Let us notice that, in the simpler case of constant returns to scale, when the variable z appears in the formulation of (D), (i.e. $z = 0$) it is possible to define an efficiency measure by $E_D = v^T y_o / u^T x_o$. In the context of input or output oriented DEA methods, this method of measuring efficiency is well known as technical efficiency. However, in the case of variable returns to scale, when the variable $z \in \mathbb{R}$ does not appear in the inequality (4) of the dual model, it is not clear how to partition z into the virtual input and virtual output. Let us remark that in the case of input oriented models, where $u^T x_o = 1$, the technical efficiency is given by $E_x = \frac{v^T y_o + z}{u^T x_o}$. In the case of output oriented models, where $v^T y_o = 1$, technical efficiency is given by $E_y = \frac{v^T y_o}{u^T x_o - z}$. Of course, here the values u, v, z may depend on the model under consideration. Since the additive model is non-oriented, it would be appropriate to use its solution (u, v, z) to define a measure which would compromise between the two extreme points of view represented by E_x and E_y. This could be made by several ways. Our intention to obtain a measure E_D with values from $[0, 1]$ led us to the following definition

$$E_D := \frac{v^T y_o + z}{u^T x_o}, \text{ if } z \geq 0, \quad E_D := \frac{v^T y_o}{u^T x_o - z}, \text{ if } z < 0.$$

Note that if both $v^T y_o$ and $u^T x_o$ are positive (which was fulfilled in the case of our data) then, by (4), $E_D \in (0, 1]$. (Let us note that the assumption just formulated follows from the stronger requirement $x_j \geq 0,\ x_j \neq 0,\ y_j \geq 0,\ y_j \neq 0,\ j = 1, \ldots, p$ which is often used as the standard assumption in DEA.)

It can be easily seen that

$$E_D = \max\left\{\frac{v^T y_o + z}{u^T x_o}, \frac{v^T y_o}{u^T x_o - z}\right\}.$$

which gives a simple interpretation of this measure as the maximum of E_x and E_y. Let us note that E_D is unit invariant but not translation invariant. We will refer to this method as the dual method.

5 Results of the DEA

In this section we discuss results of DEA obtained by the primal and dual methods for data on branch as well as subbranch offices. We compare results of different measures of efficiency defined in Section 4. A correlation analysis of the results obtained by primal and dual methods ((4.2) and (4.3)) is also presented.

5.1 Branch offices

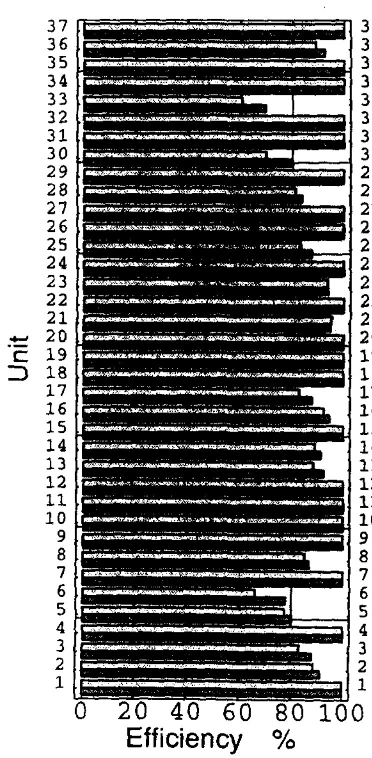

Fig. 1. Comparison of the efficiency measures E_P (dark) and E_D (light grey) for branch offices.

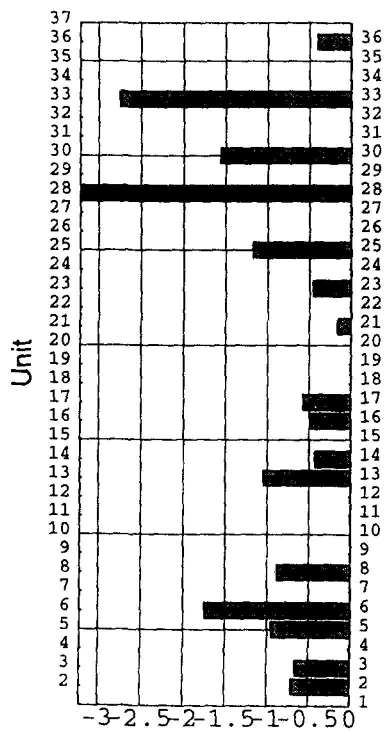

Fig. 2. Optimal value of the objective function for branch offices.

As it was already mentioned in Section 2, a key requirement of any DEA method is that the number of analyzed units should be sufficiently large compared to the number of inputs/outputs (otherwise, most units are classified as efficient).

Recall that the number of branch offices was just 37 and this is why many of branches were rated as efficient. More precisely, 21 branches were classified as efficient whereas only 16 units were inefficient (see Figures 1,2).

In Figures 3,4 below we show inputs and outputs of an inefficient unit # 28. Light grey colored bars represent reserves (slacks) in inputs/outputs computed by using the primal model. The values of inputs/outputs are scaled (0-100%) with respect to the largest values of each particular input/output. The λ vector for this unit has dominant index $\lambda_{12} = 0.92$. It means that this unit was mostly compared to the efficient unit # 12 belonging to the efficient frontier.

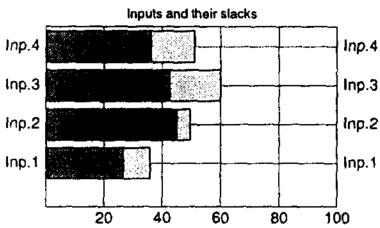

Fig. 3. Inputs for the unit #28

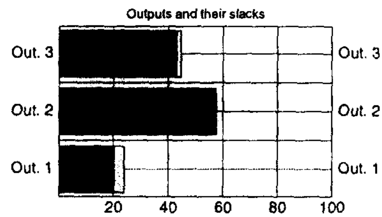

Fig. 4. Outputs for the unit #28

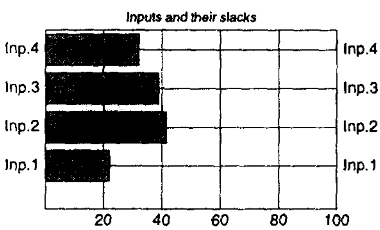

Fig. 5. Inputs for the unit #12

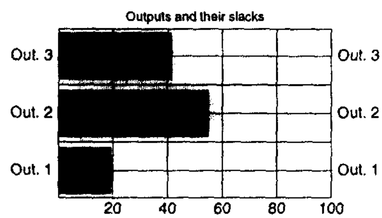

Fig. 6. Outputs for the unit #12

It is clear (Fig. 4 and 6) that the efficient unit # 12 has approximately the same outputs as unit # 28 while the inputs for # 28 are much higher compared to those of # 12. This information is of practical importance from the point of view of decision making.

5.2 *Subbranch offices*

By contrast to the network of branch offices, the total number of subbranches was very high (591) compared to the number of inputs and outputs. As it is shown in the table below this feature results in a wider range of values of efficiency for subbranches.

In Tab. 3 we present the distribution of the measures of efficiency E_P and E_D for the 591 subbranch offices. It turned out that the measure of efficiency E_D is more uniformly distributed than the measure E_P. We also present a distribution of the efficiency measure E_O based on optimal values of the objective function.

Table 3 Distribution of efficiencies E_P, E_D, E_O of subbranches.

Efficiency interval (%)	0-20	21-35	36-50	51-65	66-80	81-90	91-100	100
# of subbrns for E_O	1	0	2	8	41	41	448	50
# of subbrns for E_P	1	1	132	238	132	28	9	50
# of subbrns for E_D	59	137	149	104	65	22	5	50

The next figures show the results of primal and dual method for the set of 591 subbranch offices. The results are presented only for a subset of 20 subbranch offices.

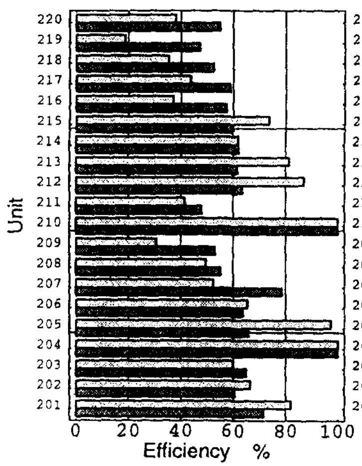

Fig. 7. Comparison of the measures of efficiency E_P (dark) and E_D (light grey) for 20 selected subbranches.

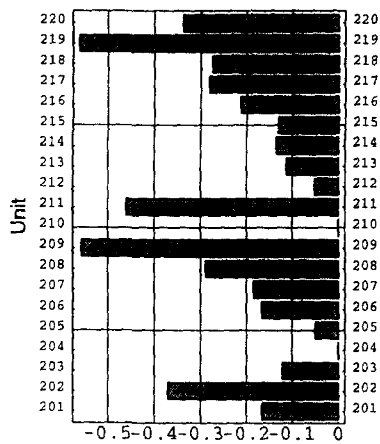

Fig. 8. Optimal value of the objective function for 20 selected subbranches.

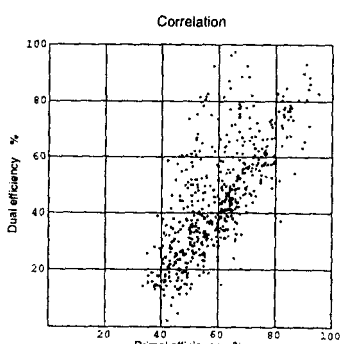

Fig. 9. The correlation between the measures of efficiency E_P and E_D.

It is obvious from Fig. 7 that, in general, the efficiency measures E_P and E_D need not have the same values. The correlation between the measures of efficiency E_P

and E_D is 84% and the relationship between them is depicted in the correlation diagram Fig. 9.

6 Discussion

6.1 Comments on the model selection

The normalized weighted additive model we have used is not the only model satisfying our requirements formulated at the end of Section 2. In this context we have to emphasize that the requirement of translation invariance of Section 2 could be weakened. In fact, in the data set of SLSP, the only negative values appeared in the output variables and, therefore, it would be sufficient to require translation invariance with respect to outputs. From the variety of the basic DEA models three models have conformed our weakened requirement. Those were the BCC input oriented model and two versions of the weighted additive model.

We first discuss the BCC input oriented model. It is well known that it is unit invariant and corresponds to variable returns to scale. Moreover, it is invariant with respect to the translations in outputs as was proved by Pastor (1996). The advantage of the model is its input orientation, a feature commonly being considered and welcomed for bank branch analysis (Schaffnit, Rosen and Paradi (1997)). Having experimented with this model we have finally not employed it in our final analysis. In addition to well known numerical problems which make its use cumbersome there was a principal reason for our decision: it is well known that the measure of efficiency defined by this model does not capture all non-zero inputs and outputs slacks. Practical experience with SLSP data showed that most of those slacks were very large and thus represented a major contribution to inefficiency.

Another option was to employ weighted additive models which, under suitably chosen weights, may be not only translation but also unit invariant. Such a model is the normalized weighted additive one presented in Section 3. In this case the weights in the objective function are reciprocal values of the sample standard deviation of the corresponding input or output variable. Another weighted additive model has been studied by Cooper, Thompson and Thrall (1996). It differs from our model by the choice of the weights: the sample standard deviation is replaced by the difference of the greatest and the smallest value of the corresponding input/output variable (of course, in order to normalize the measure the objective function has to be divided by the total number of all inputs and outputs). As proved by Cooper *et al* (1996) also this model is unit and translation invariant. An advantage of this model is that the optimal value for this model is *á-priori* bounded by -1. This model was applied to SLSP data set by Némethová (2001) and it was shown that the corresponding optimal values exhibit the same accumulation effect as the efficiency measure E_O for our model.

However, in contrary to Cooper's weighted model our normalized weighted model is less sensitive with respect to the extremal (minimal, maximal) values in particular input/output data sets. Hence omitting worst performing units would change our weights less significantly compared to Cooper's ones. Therefore we chose this model as a basis for development of measures E_P, E_D of efficiency studied in Section 4.

6.2 *Comments on the efficiency measures*

DEA not only rates efficiency but also locates the sources of inefficiency and estimates the amounts of inefficiency. However, while the concept of efficiency is (for specified returns to scale) well defined by the DEA theory and most of the models are able to decide the question of efficiency or inefficiency, the question how to measure the amount of inefficiency remains to be a subject of permanent intensive research in DEA.

In Section 4, we have proposed and analyzed three measures of efficiency which are functions of the optimal solutions to the additive model. The first measure E_O is computed from the optimal value of the model. It compares the rated unit to the efficient virtual unit by means of weighted *differences* of the virtual and real input/output values. This measure is unit and translation invariant. A major disadvantage is that the values of this measures are not distributed uniformly and most of units have their scores close to 100% (see Table 3). This measure paradoxically depends, through a choice of the scaling parameter $\varepsilon \ll 1$, on the optimal value of the worst performing unit.

On the other hand, the measures of efficiency E_P and E_D introduced in Sections 4.2 and 4.3. are computed from the optimal solutions to (P) and (D) and measure efficiency by means of weighted *ratios*. An unavoidable consequence is that they are not translation invariant and may depend on the choice of the optimal solution to (P) or (D). An advantage, at least in the case of the SLSP data, is that their resulting values are more uniformly distributed in $(0, 1)$ and are not so dramatically sensitive to the input/outputs values of the worst performing units. Furthermore, these measures can be applied to solution vectors in various other DEA models.

6.3 *Conclusion*

Because of the structure of the branch network of SLSP, in particular the large number of its subbranches, the analysis of latter turned out to be a very good test example for various DEA models and their efficiency measures. Moreover, the ratings of the branches and subbranches of SLSP based on our analysis were largely in accord with the semi-intuitive image of the management of SLSP. Except of giving a much more objective performance evaluation tool, DEA was appreciated by the management because of its transparency. In particular, for each rated unit DEA singled out a few efficient units to which the former was compared. This was found extremely valuable in our case with such a large number of DMUs in which evaluation by other, in banking practice used methods, lacks transparency.

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