1.4.1 Definition of VAR

VAR can be given the following intuitive definition.

VAR

VAR summarizes the worst loss over a target horizon with a given level of confidence.

More formally, VAR describes the *quantile* of the projected distribution of gains and losses over the target horizon. If c is the selected confidence level, VAR corresponds to the $1 - c$ lower-tail level. For instance, with a 95 percent confidence level, VAR should be such that it exceeds 5 percent of the total number of observations in the distribution.

1.4.2 Illustration of VAR

To illustrate the computation of VAR, consider, for instance, an investor who holds $100 million worth of medium-term notes. How much could the position lose over a month?

To answer this question, we simulate the 1-month return on this investment from historical data, considering only price movements. Figure 1–5 plots monthly returns on 5-year U.S. Treasury notes since 1953. The graph shows returns ranging from below 5 percent to above 5 percent.

BOX 1–3

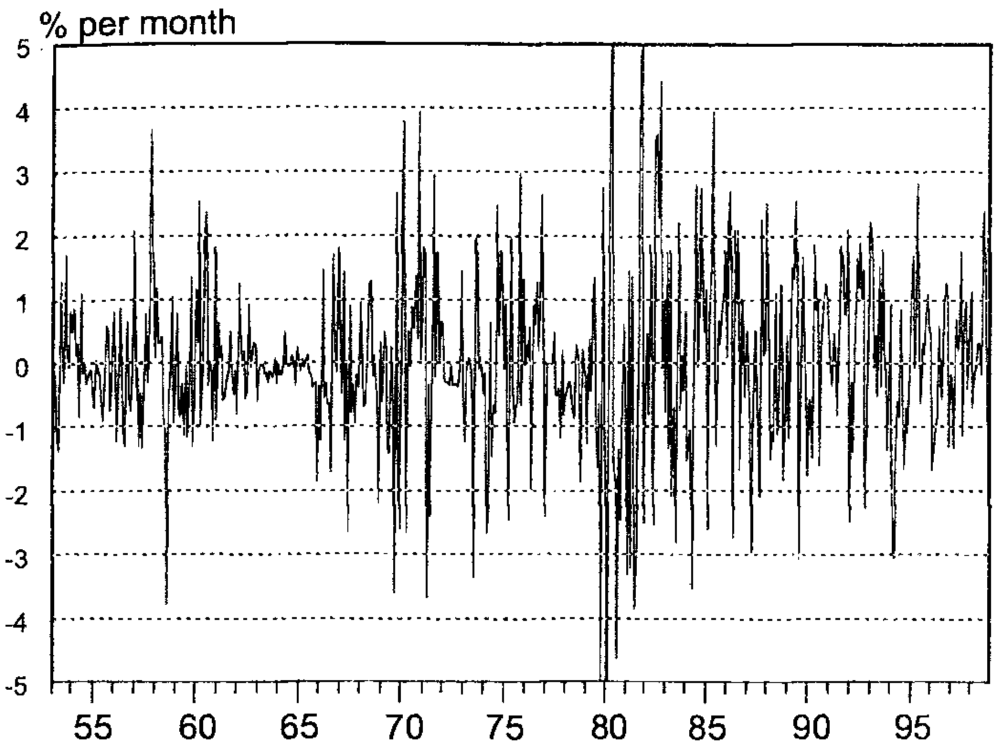
THE ORIGINS OF VAR

Till Guldimann can be viewed as the creator of the term *value at risk* while head of global research at J.P. Morgan in the late 1980s. The risk-management group had to decide whether *fully hedged* meant investing in long bonds, thus generating stable *earnings*, or investing in cash, thus keeping the market *value* constant. The bank decided that "value risks" were more important than "earnings risks," paving the way for VAR.

At that time, there was much concern about managing the risks of derivatives properly. The Group of Thirty, which had a representative from J.P. Morgan, provided a venue for discussing best risk-management practices. The term found its way through the G-30 report published in July 1993. Apparently, this was the first widely publicized appearance of the term *value at risk*.

FIGURE 1–5

Returns on medium-term bonds.



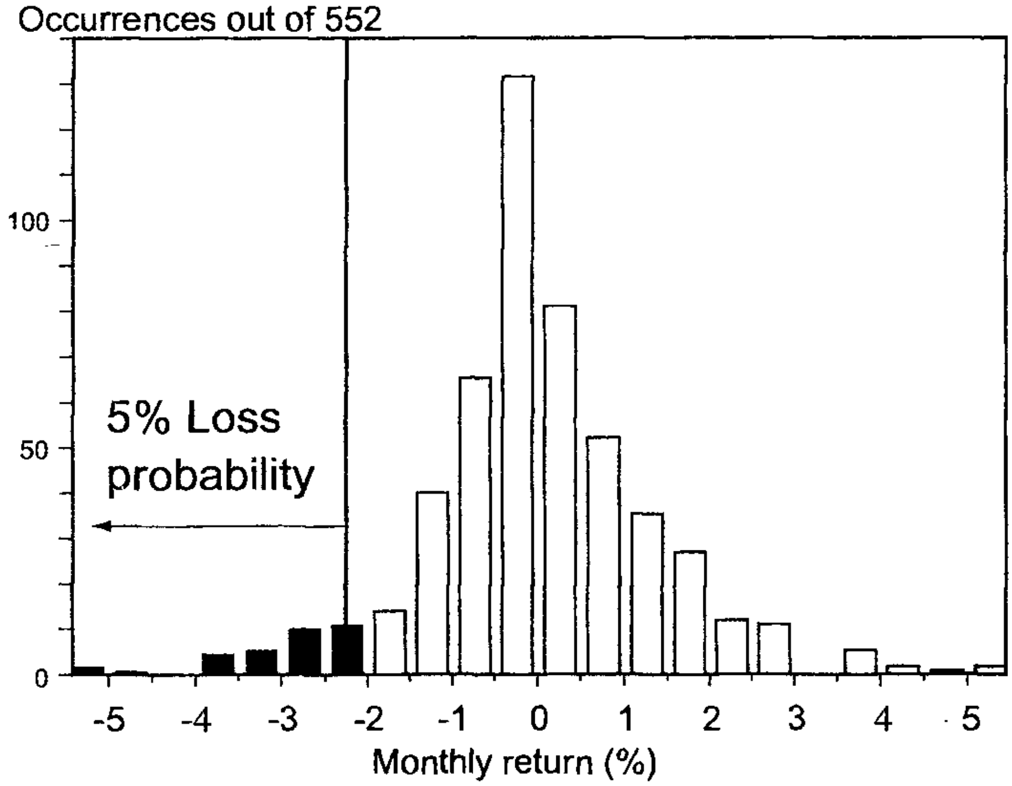
Now construct regularly spaced "buckets" going from the lowest to the highest numbers and count how many observations fall into each bucket. For instance, there are two observations below −5 percent. There is another observation between −5 and −4.5 percent. And so on. By so doing, we construct a *probability distribution* for the monthly returns, which counts how many occurrences have been observed in the past for a particular range. This *histogram,* or frequency distribution, is represented in Figure 1–6.

Next, associate with each return a probability of observing a lower value. Pick a confidence level, say, 95 percent. We need to find the loss that will not be exceeded in 95 percent of cases, or such that 5 percent of observations, that is, 27 of 552 occurrences, are lower. From Figure 1–6, this number is about −2.5 percent.

The choice of the 95 percent level is relatively arbitrary and is discussed in greater detail later. Users now report their VAR with various incompatible parameters. Assuming a normal distribution, however, it is easy to convert all these disparate measures into a common number. If the VAR number is used to assess an appropriate capital cushion, however, the confidence level should be chosen very carefully.

FIGURE 1–6

Measuring value at risk.



The choice of the holding period, 1 month or 1 day, is also relatively subjective. For a bank trading portfolio invested in highly liquid currencies, a 1-day holding period may be acceptable. For an investment manager with a quarterly rebalancing and reporting focus, a 90-day period may be more appropriate. Ideally, the holding period corresponds to the longest period needed for an orderly portfolio liquidation. A bank trading portfolio, for instance, will be much easier to close out than a portfolio invested in stocks from emerging markets. In the former case, for instance, tens of millions of dollars can be transacted in an instant; in the latter case, the same amount may take days or weeks to find willing counterparts. From the viewpoint of a regulator, the horizon should reflect the tradeoff between the costs of frequent monitoring and the benefits of early detection of potential problems.

We are now ready to compute the VAR of a $100 million portfolio. Based on the preceding analysis, we are 95 percent confident that the portfolio will fall by no more than $100 million times −2.5 percent, or $2.5 million, over a month. Hence the value at risk is about $2.5 million. A similar result would have been obtained by taking the standard deviation of the historical series, which is 1.5 percent, and multiplying it by the 95 percentile of the standard normal distribution, which is 1.645. The result from this normal model is rather close, at $2.4 million.

The market risk of this portfolio can now be communicated effectively to a nontechnical audience with a statement such as this: *Under normal market conditions, the most the portfolio can lose over a month is about $2.5 million at the 95 percent confidence level.*

1.5 VAR AND THE EVOLUTION OF RISK MANAGEMENT

VAR is the latest step in the evolution of risk-management tools. Consider, for instance, a fixed-income portfolio whose value is a function of the current yield.[5]

Figure 1–7 describes the classic risk-management approach. The first step is a valuation problem, which involves solving for the price given the current yield. To understand risk, one could approximate movements in the price through a sensitivity measure. This leads to the concept of duration, which measures the linear exposure of the position to interest rate risk. This approximation can be refined further by convexity, which measures the quadratic, or second-order term. All these are local approximations. Another approach is scenario analysis, which values the portfolio for a series of interest rates, using full valuation.

VAR goes one step further, though. It combines this price-yield relationship with the probability of an adverse market movement. This is shown in Figure 1–8, which describes how the price function is combined with a probability distribution for yields to generate a probability distribution for the bond price. Thus VAR describes the *probability boundary* of potential losses.

VAR is much broader than this simple example, though. Besides interest rates, it can encompass many other sources of risks, such as foreign currencies, commodities, and equities, in a consistent fashion. VAR accounts for leverage and correlations, which is essential when dealing

5. See, for instance, Golub and Tilman (2000) for systematic applications to the fixed-income markets.