# Convex Optimization - Lecture Syllabus

**Course Summary:**

Convex optimization is a one semester course for students with Bachelor degree in applied mathematics. The students are required to be familiar with the basic techniques of unconstrained optimization as well as the basic theory of linear and nonlinear programming.

Convex optimization is an advanced course about a special class of mathematical programming, which became very popular within the last decades. It finds many applications in areas such as control systems, signal processing, networks, optimal design, data analysis, modeling, statistics and finance. It is very important to recognize a problem as convex because then it can be efficiently solved via interior point methods. The course offers a survey of convex analysis results, convex optimization duality theory, many applications examples and introduction to interior point methods for convex programming.

**Lecturer:**
Mária Trnovská

**Textbooks and study materials:**

- S. Boyd, L. Vandenberghe: *Convex Optimization*, Cambridge University Press
- A. Ben-Tal, A. Nemirovski: *Lectures on Modern Convex Optimization*, MPS-SIAM Series on Optimization, SIAM
- R.T. Rockafellar: *Convex Analysis*, Princeton University Press
- S. Boyd: *Additional Exercises for Convex Optimization*
  https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook_extra_exercises.pdf

**Grading:**
Final grade is determined by project, homeworks and presentation. Students are allowed to work in groups. The project work consists of practical part that includes matlab programming and presentation of the project. The homeworks solutions as well as the projects are written in Latex and submitted as a pdf file, the matlab code is included. Homeworks have weight of 40%, the project has a weight of 50%, the presentation has a weight of 10%.
**Grading scale:**

| % | |
|---|---|
| 100-90 | A |
| 89-78 | B |
| 77-65 | C |
| 66-53 | D |
| 52-38 | E |

Course information and lecture slides are available at:
http://www.iam.fmph.uniba.sk/institute/trnovska/myteaching.html

## Lecture Topics

**Topic 1: Convex optimization – what is it an what is ot good for?**
Importance of convex optimization – using of convex optimization – history of convex optimization – the basic convex optimization problem formulation – important classes of convex optimization – generalized problem – example: semidefinite programming and its application in portfolio optimization

**Topic 2: Convex analysis – convex sets**
Basic types of convex sets and their formal expressions – operations that preserve convexity – Supporting hyperplane theorem and proof, Separating hyperplane theorems and proofs

**Topic 3: Convex analysis – convex functions**
Definition and basic properties of convex functions – first and second order convexity conditions – epigraph – operations that preserve convexity – quasiconvex functions - first and second order quasiconvexity conditions – operations that preserve quasi convexity – strongly convex functions – first and second order conditions for strong convexity – convexity with respect to generalized inequialities

**Topic 4: Convex optimization problems**
Convex optimization problem in standard form – feasibility problem – formulation of equivalent problems – local and global optima – optimality condition for differentiable functions – solving quasiconvex problem via convex feasibility problems – generalized convex optimization problem – known classes of convex optimization and their relation – geometric programming

**Topic 5: Duality** Lagrangian and Lagrange dual function – Lagrange dual problem – weak duality – Slater condition – strong duality – Slater theorem and proof – saddle point interpretation – criteria of suboptimality – examples: minimum volume covering ellipsoid, entropy maximization, nonconvex quadratic problem – theorems of alternatives – generalized duality and semidefinite programming example

**Topic 6: Optimality conditions** Karush-Kuhn-Tucker optimality conditions – example: water filling – perturbations – local and global sensitivity analysis

**Topic 7: Applications of convex optimization** Markowitz portfolio via quadratic programming, semidefinite programming, bicriterial optimization, second-order cone programming, robust approach – signal reconstruction – geometric applications – linear and nonlinear separation, minimum volume ellipsoid – optimal design of experiments – CVX implementation

**Topic 8: Interior point methods** Basic idea of interior point methods - logarithmic barrier and central path – barrier method – Newton method – stopping criteria – choice of parameters – primal-dual interior point method – selfconcordance property